A Study of Prime Sextuplet Analysis Using Gestalt Approach

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23.04.2021
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1 Additions since original publication

Since this is a work in progess with "rolling releases", this section will point to the changes made since the study's initial publication. Readers who are updating can find the new information by comparing the "latest additions" date above.

- 23.04.2021:
  An even simpler alternative algorithm for getting a Q's offset on page 38
  Critical sections, between families, General Analysis 6.2 on page 65.
  A description of the infinite sextuplet problem 6.3 on page 73.

- 13.03.2021:
  Alternative algorithm for getting a Q's offset using arithmetic modulo on page 37
  Critical sections, within family, expanded. on page 63
  A new diagram regarding family relationsips to TNumbers and to each other, on page 27

2 Introduction

Following an interest in prime sextuplets and whether this prime "edge-case" could provide hints as to whether the number of prime sextuplets is infinite, I found structures that could provide clues and which provide possibilities for analysis.

Even though the focus for my interest was/is sextuplets, the structures make themselves immediately amenable to quintuplet and quadruplet analysis also, almost automatically. The primary focus, unless otherwise noted, will be sexuplets, but quints and quads are also included here. I will occasionally refer to the word "Tuplets" which refers in general to sextuplets, quintuplets, and quadruplets as a group.

All the structures described here were first worked out on paper, and then, as proof of concept, written into computer code. The structures, and associated equations, worked and 100% accurate lists of Tuplets can be generated. The computer code is not the most efficient, nor the fastest, but it does prove that the methods and structures in this document are correct. The code and app can be found at: https://github.com/Juuliuus/juusprime.

The work focuses on blocks of numbers which fall naturally out of the primes. The primes are divided into three major groups each handled differently. Rather than primality testing, it uses the “effect” a prime group will have at any particular point in the number line.

3 Preface

My education is in Biology, and I am only familiar with the usual mathematics associated with such a study. I have been very careful to check all the work and equations and structures for correctness, and, as mentioned, have put these into play in a proof of concept computer program. There are areas where I see possibilities for potentially insightful analysis that are, at this time, mathematically beyond me. I will point these out and perhaps a more mathematically minded person will find them of interest and be able to solve them.

I may mention things here that are already common knowledge to more mathematically inclined people. If that is the case, I apologize, and do hope you will find something new that can picque your interest.

I will also go into more detail than would be otherwise be normal in a mathematically themed paper because I anticipate that my audience, if any, will be primarily not professional mathematicians, and so I seek to be clear and readable, and avoid confusion. And some details are simply reminders for me.

This document was written with the help of the LyX LaTeX editor.

4 Initial Assertions

Before getting to the description of how the three groupings of the primes numbers used to develop this prime sextuplet study are built, there are some observations about primes that will be of use in said development.

4.1 The “effective” start of a Prime’s influence

First principles for this “effective” start come from a study of the simple sieve of Erathosthenes.

This assertion is that any prime number will not be “effective” at clearing a sieve until we reach that prime number’s value squared. This means we don’t need to consider a prime’s effects until its square.

We show this by applying primes piecemeal. For instance consider the sieve in fig. 1. We first apply only the prime 2. When we do, we find the sieve tells us that all odd numbers

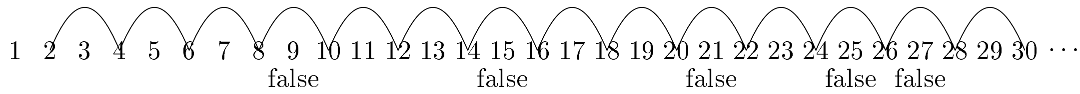


Figure 1: Sieve of Eratosthenes with only 2 applied

are "prime". That is, there are many "false" primes remaining. However also note that at 2's square, 4, the primes are accurately sieved up to 9.

Now apply 3, as in fig. 2, and many of these false primes are removed. All, in fact, between 9 (3^2) and 25 (5^2).

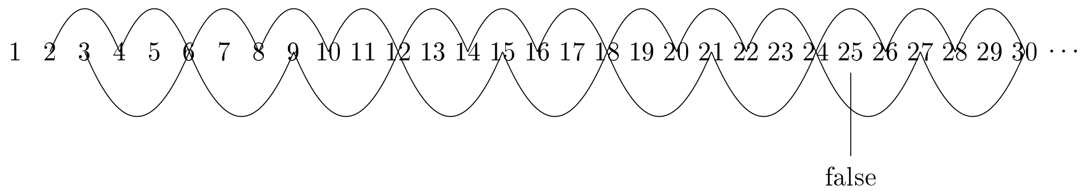
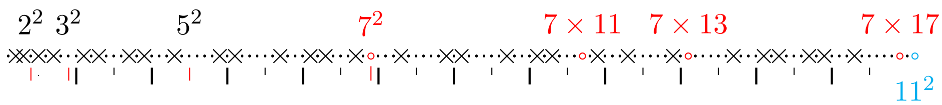


Figure 2: Sieve of Eratosthenes with only 2 and 3 applied

Finally we apply the primes 2, 3, and 5 and see clearly that a prime has no effect on the sieve until its square. Prime 7, which has not been applied yet, has absolutely no effect on anything less that 49, its square. Were we now to apply 7, the false primes of 77, 91, and 119 would disappear, but 121 (11^2) remains falsely shown as "prime". Similarly, Prime 11 has no effect and does not help in identifying primes less than 121.



This is what is meant by primes have no effect/influence until their squares. This will be of importance when we construct the prime structures for 31-59.

4.2 Interval-izing regions between squares

It will be of use to us to be able to compare regions between squares of primes based on a chosen interval size. While this is certain to be derived in some mathematical text, I have no reference for it, and so re-derive it here. This will be of use for prime structures 31-59.

This assertion is not limited to primes but applies to any pair of numbers, and in this section it will be generalized for any number n.

The difference between two numbers is exactly a factor of the distance between their squares. One can choose this difference, or interval, as one likes.

For example, assume we want, for whatever reason, to construct comparable regions for squares for an interval size of 8. Pick any number > 0 and add our interval to it. Let's say we pick 7 as our number, so we add 8 to it for our next number: 15. The region between 7^2 and 15^2 will be exactly divisible by 8; that is, that region is divided up into some number of intervals of size 8.

Were we to pick 25 as our number, we add 8 to get 33. We also find that the region between those numbers squared will also be evenly divisible by 8.

In general, with p being the initial number we choose, and C being the interval we want, our second number will be $p + C$. Taking the difference of their squares and dividing by C will give us x (the number of intervals of C length in the region between their values squared). We start with p and $p + C$:

$$x = \frac{(p + C)^2 - p^2}{C}$$
$$x = \frac{p^2 + 2pC + C^2 - p^2}{C}$$
$$x = \frac{2pC + C^2}{C}$$
$$x = 2p + C \qquad (1)$$

$C \neq 0; p \geq 0$

Checking: $7^2 = 49$ and $15^2 = 225$. Taking the difference: $225 - 49 = 176$ and dividing by 8: $176/8 = 22$.
Using the formula with $p = 7$ and $C = 8$: $2(7) + 8 = 22$

So the region between these squares has 22 sections each of which has a size of 8.

We also want to extend this relationship as a series from our initial number p $(= 7)$ each determined by our interval $(= 8)$. We can generalize the equation for any member of our series by calculating the range from $p + Cn$ to $p + C(n + 1)$, which expands to $p + Cn + C$:

$$x = \frac{(p + Cn + C)^2 - (p + Cn)^2}{C}$$
$$x = \frac{p^2 + 2pCn + 2pc + 2C^2n + C^2 + C^2n^2 - p^2 - 2pCn - C^2n^2}{C}$$
$$x = \frac{2pC + C^2 + 2C^2n}{C}$$
$$x = 2p + C + 2Cn \qquad (2)$$

$C \neq 0; p \geq 0$; $n \geq 0$; we see that equation (1) is the special case of equation (2) where $n = 0$. Note that after an initial selection of starting number, p, then $2p + C$ becomes constant.

As regards our example series that begins with 7 and uses intervals of 8: n corresponds to the 0 based cardinal value of the member of the series. The first of of this (zero based) series is 7-15 ($n = 0$), the 2nd is 15-23 ($n = 1$), the 3rd 23-31 ($n = 2$), and so on. Using the 3rd of our example series setting $p = 7$; $C = 8$; n=2:

Checking: $2(7) + 8 + (2 \times 8 \times 2) = \frac{(31^2 - 23^2)}{8} = 54$

5 Prime structures

When the prime sextuplets were analyzed as regards their formation and "survival", it became clear that the primes could be arranged into groups that would facilitate sextuplet study.

There are three groupings of primes that are here considered "natural":

- Primes 2,3,5
- Primes 7-29
- Primes 31-59

There are also indications that the 3rd group has other natural regions that may be worthy of study.

The idea is to set up the number line structure into groups that can be checked for Tuplets by block addition, in effect a Gestalt, or pattern, check. This means that no numbers are analyzed to see if they are prime at all. In essence, all 6 members of the sexuplet (and tuplet) structures are checked at once. After developing this idea one finds order in the primes that was not before apparent, order that can be quantified, to a good extent, in equations.

5.1 Primes 2, 3, & 5: Template Block

The formation seed of sextuplets is easily seen when we return to the partial sieve of Erathosthenes of 2/3/5. As shown in fig. 3 they pop out naturally from, what I call, the Prime Template:

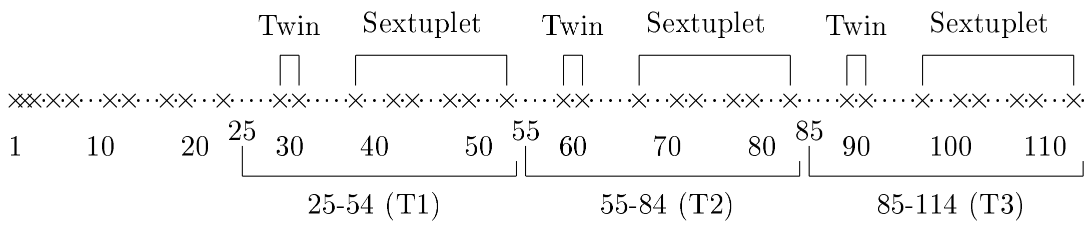


Figure 3: Formation of Templates via 2/3/5 prime application

This partial sieve divides the number line up into repeating groups of size 30, starting at 25. They are numbered starting from 1 (the 0th Template starts at -5) at Template Number 1 (or T1 or T_1). The first Template runs from 25 to 54, inclusive, and so on. All Templates begin with a number ending in 5. This study focuses on Sexuplets and Tuplets, so the “rogue” twin primes in the left portions of each Template are ignored. If a number in parenthesis follows a TNumber it is showing the starting counting number at that TNumber, for example: T_1 (25) means T_1 starts at 25.

Figure 4 shows T_3 where the first post T_0 Sexuplet resides (97, 101, 103, 107, 109, 113), detailing its alignment with the Template. This figure also indicates the mod and counting offsets of the positions inside a Template.

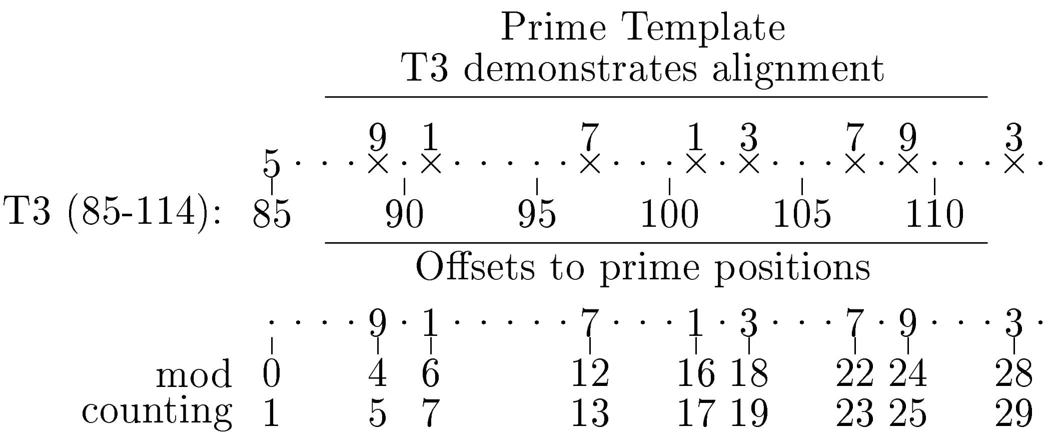


Figure 4: Prime Template detail

Several things are apparent after seeing the Template: sextuplets must always start with a prime number that ends with a 7. Twin primes will only be found with primes ending in x9/x1, x1/x3, x7/x9, no other ending digits are possible for twins. The twin x9/x1 can never be a remnant of a sextuplet.

Since every block starts out with a pristine Sextuplet, they can only ever be altered or destroyed by subsequent primes, they can never be built up. One sees immediately how the other tuplets are formed: quintuplets are formed if one or the other of the end primes of the Sextuplet are removed. If both ends are removed the remant will be a quadruplet. If any of the primes of the quadruplet portion are removed (turning the remant into a broken Sexuplet or broken quintuplet, or into triplets) I consider the Tuplet to be destroyed for the purposes of this study.

For the most part, from here on (unless otherwise noted) number lines will represent TNumbers, not counting numbers. To convert counting numbers (N) to TNumbers (T) see equation (3), for the reverse see equation (4):

$$T = \left\lfloor \frac{N+5}{30} \right\rfloor = (N+5)\,\mathrm{div}30 \tag{3}$$

$$
\begin{aligned}
N_{\text{start}} &= (T \times 30) - 5 \\
N_{\text{end}} &= N_{\text{start}} + 29 = (T \times 30) + 24
\end{aligned}
\tag{4}
$$

What we can finally say about the primes 2/3/5 is that there is no need to worry about them further. They have given us a template to use which always starts off with a pristine Sextuplet. These primes come no farther into the calculations and structures that follow.

5.2 Conventions

5.2.1 Symbols for template diagrams and prime structures

Symbols are often used in files and diagrams to symbolize the Tuplet result at any particular Template:

- ├─┤: Sextuplet. The two vertical bars represent the end primes of the sextuplet, the horizontal the quadruplet (or double-twin).
  In the computer code it is represented by 0.
- ├─ : Left-sided Quintuplet.
  In the computer code it is represented by 1.
- ─┤ : Right-sided Quintuplet.
  In the computer code it is represented by 2.
- – : Quadruplet.
  In the computer code it is represented by 3.
- X : Tuplet is destroyed.
  In the computer code it is represented by 4.
- o : Spacer; i.e., a skip or inflationary space is added in the diagrams
- ● : equivalent to a Sexuplet.

The symbol ● is a less busy equivalent to ├─┤. When you see either of these symbols in a table, diagram or file it means the symbol's effect is that the Sextuplet in that template survives and passes on to the next check.

5.2.2 Standards

- C, equal to 30, is the constant size of the length of the Prime Template.
- T_n is any TNumber
- T_{exp} stands for TNumber expanded (which is done using equation 4 on this page).
- forms of $T_n\,(N)$ show the TNumbers starting value in the parentheses; e.g. $T_1\,(25)$

5.3 Primes 7 through 29

I believe one word describes the primes 7-29 situation: chaos. Is it possible to find a "natural" pattern here that would be of use in studying Sextuplets?...

We now have a tool, the Prime Template, we can use to check the status of our starting pristine Sextuplet. The need is simple: starting at any TNumber we need to apply the effects of the seven primes 7, 11, 13, 17, 19, 23, and 29 at that Template. If the effect of the prime at that Template is a Sextuplet (meaning a sextuplet is still allowed, it is allowed to "pass") at each of these primes then the original pristine sextuplet is allowed; i.e., it survives. If any of the effects is a quintuplet, then the original sextuplet is altered to be a quintuplet. Likewise, if the effect of any is a quadruplet the sextuplet is altered to be a quadruplet (see section 5.4 on page 23).

So our first consideration is simple: we need some structure that can tell us what effect a prime will have at any particular Template Number. We need to map the effects of these primes into a structure that can be used to determine the prime's effects at a TNumber. We will first analyze 7 to build up its "natural progression".

Both equations and computer code are simpler to code using mod results (0 based counting), and it is preferred in this study to use mod crossings and positions.

Many of the details shown in the Prime 7 initial section will also apply to all the other primes 11-29.

Each prime 7-29 we will analyze has these useful properties: its value, its value squared, and the TNumber in which it becomes effective (see section 4.1 on page 4).

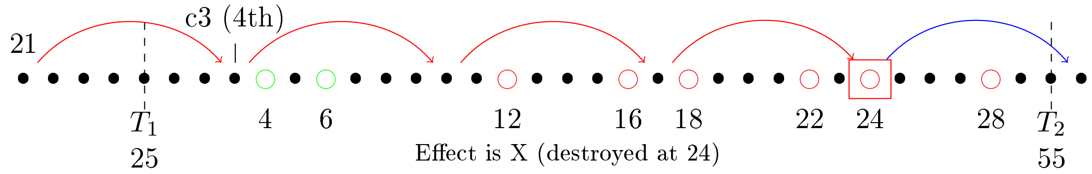
5.3.1 Prime 7

Its value is 7 and it becomes effective at $7^2 = 49$. So, evaluating 49 in equation 3 on page 8 gives us 7's effective TNumber: T_1 (25).

We now only need to determine 7's effects as it passes through T_1 and the next 6 Templates, T_1 (25) to T_7 (205), inclusive, because after that its effects will repeat. To do this we map out crossing effects against the Prime Template.

We build the crossing numbers and effects by following 7 as it passes through the Templates. By convention the crossing numbers are named after the position the prime lands on after crossing the Template border.

In the diagram below the dotted vertical lines denote the beginnings of Templates. All positions in the template are mod based. The "rogue twin" at the left (positions 4 and 6) are green colored; they are ignored in this Sextuplet study. If the prime, in this case 7, hits one of the Sexuplet positions it is emphasized with a surrounding box. The ends of the arrows point to the strike point. For clarity the beginnings of the arrows start from the last strike point, but they are not inclusive in the counting, showing that they span the next, in this case, 7 positions. Finally, the effect (See Symbol summary Symbols for template diagrams and prime structures on page 9) of this crossing is shown.



So, from this particular crossing of 7 we now know that if we find, at any TNumber we like, that the crossing number of 7 is c3, we know that the sexuplet at that TNumber is destroyed. A complete analysis of 7's movements through Templates gives us a simple look up table for effects, and a chart of its "natural progression" through the TNumbers.

For those interested in seeing the complete prime 7 crossing number analysis see Complete crossing number analysis for prime 7 on page 76.

5.3.2 Prime 7: LookUp Table and Natural Progression

As you inspect 7's crossing numbers and effects in the table below you see that out of the 7 possibilities for 7, only 1 (c6) allows a sextuplet to survive, meaning only every 7th Template ever has a possiblity to ultimately produce a sextuplet.

This shows that the difference between TNumbers that contain sextuplets, or differences of corresponding members of differing sextuplets, will always be evenly divisible by 7. This, then, shows that starting from T_3 only every 7th Template has a possibility of producing a Sextuplet: T_3, T_{10}, T_{17} etc.

Given that the closest two Sextuplets can be to each other, by Template Number, is 7 then it follows that a "twin sextuplet" must be of the form T_n and T_{n+7}. See section Twin Sextuplets on page 23 for more information about twin Sextuplets.

Prime 7 effect table:

7 CrossNum	Effect	Next Crossing
c0	┣	→c5
c1	X	→c6
c2	X	→c0
c3	X	→c1
c4	X	→c2
c5	┫	→c3
c6	●	→c4

●○○○○○

Prime 7 natural progression:

Prime 7	c3	c1	c6	c4	c2	c0	c5	… c3 →∞
Effect	X	X	●	X	X	┣	┫	…
T_n	T_1	T_2	T_3	T_4	T_5	T_6	T_7	…
T_{exp}	25	55	85	115	145	175	205	…
	0	1	2	3	4	5	6	

5.3.3 Finding Crossing Numbers

While it might seem that a simple mod function will be all we'll need to determine crossing points at Template boundaries for primes 7-29, it is not the case. In order to match the naming convention for crossing numbers, which we will use for all the prime structures we build, we need a special form; a form that will give us back "3" to match our name of c3. In equations 5 and 6, p is the value of the prime in question, T_{exp} stands for TNumber expanded (which is done using equation 4), T_n is any TNumber greater than 0, and CN, crossing number, is returned in 0 based mod position:

$$\text{CN} = (p - (T_{\text{exp}} \mod p)) \mod p \tag{5}$$

$$\text{CN} = (p - ((T_n \times 30) - 5) \mod p) \mod p \tag{6}$$

Testing for the crossing number for 7 at T_1:
$(7 - 25 \mod 7) \mod 7 = 3$
$(7 - ((1 \times 30) - 5) \mod 7) \mod 7 = 3$

5.3.4 Prime 11

This prime becomes effective at $11^2 = 121$, and so 11's effective TNumber is T_4 (115).

There is one example diagram for 11 in section Crossing map for 1 cycle of prime 11 on page 78. These example diagrams should suffice if you want to reproduce the crossing numbers and natural progressions for the primes 7-29.

Prime 11 effect table:

11 CrossNum	Effect	Next Crossing
c0	X	→c3
c1	⊣	→c4
c2	X	→c5
c3	●	→c6
c4	●	→c7
c5	X	→c8
c6	⊢	→c9
c7	X	→c10
c8	●	→c0
c9	●	→c1
c10	●	→c2

○○○○●○

Prime 11 natural progression:

Prime 11	c6	c9	c1	c4	c7	c10	c2	c5	c8	c0	c3
Effect	⊢	●	⊣	●	X	●	X	X	●	X	●
T_n	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}
T_{exp}	115	145	175	205	235	265	295	325	355	385	415
	0	1	2	3	4	5	6	7	8	9	10

5.3.5 Prime 13

This prime becomes effective at $13^2 = 169$, its effective TNumber is T_5 (145).

Prime 13 effect table:

13 CrossNum	Effect	Next Crossing
c0	●	→c9
c1	●	→c10
c2	⊢	→c11
c3	X	→c12
c4	●	→c0
c5	X	→c1
c6	●	→c2
c7	●	→c3
c8	●	→c4
c9	X	→c5
c10	●	→c6
c11	X	→c7
c12	⊣	→c8

●○○○○○

Prime 13 natural progression:

Prime 13	c11	c7	c3	c12	c8	c4	c0	c9	c5	c1	c10	c6	c2
Effect	X	●	X	⊣	●	●	●	X	X	●	●	●	⊢
T_n	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}
T_{exp}	145	175	205	235	265	295	325	355	385	415	445	475	505
	0	1	2	3	4	5	6	7	8	9	10	11	12

5.3.6 Prime 17

This prime becomes effective at $17^2 = 289$, its effective TNumber is T_9 (265).

Prime 17 effect table:

17 CrossNum	Effect	Next Crossing
c0	●	→c4
c1	X	→c5
c2	●	→c6
c3	●	→c7
c4	●	→c8
c5	X	→c9
c6	●	→c10
c7	X	→c11
c8	●	→c12
c9	●	→c13
c10	●	→c14
c11	⊢	→c15
c12	⊣	→c16
c13	●	→c0
c14	●	→c1
c15	●	→c2
c16	X	→c3

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Prime 17 natural progression:

P 17a	c7	c11	c15	c2	c6	c10	c14	c1	c5	c9	c13	c0	→17b
Eff.	X	⊢	●	●	●	●	●	X	X	●	●	●	→
T_n	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}	T_{19}	T_{20}	→
$T_{\exp}$	265	295	325	355	385	415	445	475	505	535	565	595	→
	0	1	2	3	4	5	6	7	8	9	10	11	

17b	c4	c8	c12	c16	c3
...	●	●	⊣	X	●
...	T_{21}	T_{22}	T_{23}	T_{24}	T_{25}
...	625	655	685	715	745
	12	13	14	15	16

5.3.7 Prime 19

This prime becomes effective at $19^2 = 361$, its effective TNumber is T_{12} (355).

Prime 19 effect table:

19 CrossNum	Effect	Next Crossing
c0	●	→c8
c1	●	→c9
c2	●	→c10
c3	X	→c11
c4	●	→c12
c5	X	→c13
c6	●	→c14
c7	●	→c15
c8	●	→c16
c9	⊢	→c17
c10	●	→c18
c11	●	→c0
c12	⊣	→c1
c13	●	→c2
c14	●	→c3
c15	●	→c4
c16	X	→c5
c17	●	→c6
c18	X	→c7

●○○○○○

Prime 19 natural progression:

P 19a	c6	c14	c3	c11	c0	c8	c16	c5	c13	c2	c10	c18	→19b
Eff.	●	●	X	●	●	●	X	X	●	●	●	X	→
T_n	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}	T_{22}	T_{23}	→
T_{exp}	355	385	415	445	475	505	535	565	595	625	655	685	→
	0	1	2	3	4	5	6	7	8	9	10	11	

19b	c7	c15	c4	c12	c1	c9	c17
...	●	●	●	⊣	●	⊢	●
...	T_{24}	T_{25}	T_{26}	T_{27}	T_{28}	T_{29}	T_{30}
...	715	745	775	805	835	865	895
	12	13	14	15	16	17	18

5.3.8 Prime 23

This prime becomes effective at $23^2 = 529$, its effective TNumber is T_{17} (505).

Prime 23 effect table:

23 CrossNum	Effect	Next Crossing
c0	●	→c16
c1	X	→c17
c2	●	→c18
c3	●	→c19
c4	●	→c20
c5	$\vdash$	→c21
c6	●	→c22
c7	●	→c0
c8	●	→c1
c9	●	→c2
c10	●	→c3
c11	●	→c4
c12	$\dashv$	→c5
c13	●	→c6
c14	●	→c7
c15	●	→c8
c16	X	→c9
c17	●	→c10
c18	X	→c11
c19	●	→c12
c20	●	→c13
c21	●	→c14
c22	X	→c15

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Prime 23 natural progression:

P 23a	c1	c17	c10	c3	c19	c12	c5	c21	c14	c7	c0	c16	→23b
Eff.	X	●	●	●	●	$\dashv$	$\vdash$	●	●	●	●	X	→
T_n	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}	T_{22}	T_{23}	T_{24}	T_{25}	T_{26}	T_{27}	T_{28}	→
$T_{\exp}$	505	535	565	595	625	655	685	715	745	775	805	835	→
	0	1	2	3	4	5	6	7	8	9	10	11	

23b	c9	c2	c18	c11	c4	c20	c13	c6	c22	c15	c8
...	●	●	X	●	●	●	●	●	X	●	●
...	T_{29}	T_{30}	T_{31}	T_{32}	T_{33}	T_{34}	T_{35}	T_{36}	T_{37}	T_{38}	T_{39}
...	865	895	925	955	985	1015	1045	1075	1105	1135	1165
	12	13	14	15	16	17	18	19	20	21	22

5.3.9 Prime 29

This prime becomes effective at $29^2 = 841$, its effective TNumber is T_{28} (835).

Prime 29 effect table:

29 CrossNum	Effect	Next Crossing
c0	●	→c28
c1	●	→c0
c2	●	→c1
c3	●	→c2
c4	●	→c3
c5	●	→c4
c6	●	→c5
c7	●	→c6
c8	●	→c7
c9	●	→c8
c10	●	→c9
c11	●	→c10
c12	⊣	→c11
c13	●	→c12
c14	●	→c13
c15	●	→c14
c16	X	→c15
c17	●	→c16
c18	X	→c17
c19	●	→c18
c20	●	→c19
c21	●	→c20
c22	X	→c21
c23	●	→c22
c24	X	→c23
c25	●	→c24
c26	●	→c25
c27	●	→c26
c28	⊢	→c27

○○○○●○

Prime 29 natural progression:

P 29a	c6	c5	c4	c3	c2	c1	c0	c28	c27	c26	c25	c24	→29b
Eff.	●	●	●	●	●	●	●	⊢	●	●	●	X	→
T_n	T_{28}	T_{29}	T_{30}	T_{31}	T_{32}	T_{33}	T_{34}	T_{35}	T_{36}	T_{37}	T_{38}	T_{39}	→
T_{exp}	835	865	895	925	955	985	1015	1045	1075	1105	1135	1165	→
	0	1	2	3	4	5	6	7	8	9	10	11	

29b	c23	c22	c21	c20	c19	c18	c17	c16	c15	c14	c13	→29c
...	●	X	●	●	●	X	●	X	●	●	●	→
...	T_{40}	T_{41}	T_{42}	T_{43}	T_{44}	T_{45}	T_{46}	T_{47}	T_{48}	T_{49}	T_{50}	→
...	1195	1225	1255	1285	1315	1345	1375	1405	1435	1465	1495	→
	12	13	14	15	16	17	18	19	20	21	22	

29c	c12	c11	c10	c9	c8	c7
...	⊣	●	●	●	●	●
...	T_{51}	T_{52}	T_{53}	T_{54}	T_{55}	T_{56}
...	1525	1555	1585	1615	1645	1675
	23	24	25	26	27	28

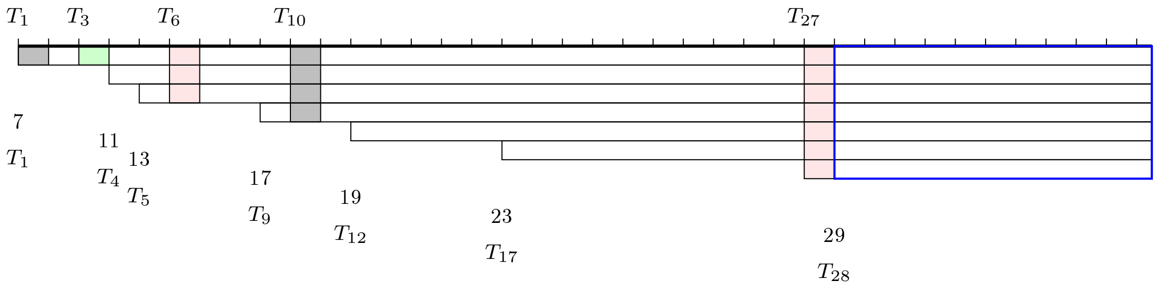
5.3.10 Usage of primes 7-29

In their most basic form they provide lookups to determine the effect each of these primes have on any Template you pick out of the infinitely many Templates there are. We will see shortly, in section Prime Basis below, how to reduce the range we choose from and how remove primes 7-29 from any further consideration in this study.

In the diagram the thick black line at the top represents the primes 2,3,5 structure. It is a line consisting of TNumbers going from $T_1 \to T_\infty$. The spacing of vertical dashes on this line represents the 30 numbers contained within each Template. Each of the primes 7-29 is shown at each's effective starting TNumber.

The diagram is read from top to bottom, so each TNumber begins at the 2,3,5 layer meaning each is a pristine Sextuplet. The shaded areas show which primes are effecting that TNumber: T_1 and T_3 are affected by 7, T_6 is affected by 7-13, T_{10} is affected by 7-17, and T_{27} is affected by 7-23. Gray areas denote destroyed sextuplets, green is a sextuplet, and the pinks are quadruplets.

We need to calculate the crossing number of the respective effecting prime (see Finding Crossing Numbers on page 12), look up the effect at that crossing number from the tables above, and then do "symbol math" (see Tuplet Math on page 23).



The crossing number of 7 at T_1 is c3. 7's c3 effect is "X", meaning it destroys the sextuplet that comes from above. All the other Templates in the diagram, outside of T_3, T_6, T_{27}, are also destroyed, or "X"d, by one or another of the primes effecting them.

The crossing number of 7 at T_3 is c6. 7's c6 effect is $\bullet$, equivalent to $\vdash\!\dashv$. It allows the sextuplet to pass, and hence the sextuplet 97, 101, 103, 107, 109, 113 survived and resides in T_3.

For the more complex analysis of the larger TNumbers we will summarize the results in a running cumulative-result table.

For T_6:

T_6	CrossNum	Effect at CrossNum	Running Result
T_6 starts as		$\vdash\!\dashv$	
prime 7 effect	0	$\vdash$	$\vdash$
prime 11 effect	1	$\dashv$	-
prime 13 effect	7	$\bullet$	-

The starting sextuplet at T_6 (175) is altered to be the quadruplet: 191, 193, 197, and 199.

Next for an example of sextuplet destruction T_{10}:

T_{10}	CrossNum	Effect at CrossNum	Running Result
T_{10} starts as		$\vdash\!\dashv$	
prime 7 effect	0	$\bullet$	$\vdash\!\dashv$
prime 11 effect	2	X	X

The sextuplet at T_{10} is destroyed by prime 11, no need to test primes 13 or 17.

And finally for T_{27}:

T_{27}	CrossNum	Effect at CrossNum	Running Result
T_{27} starts as		$\vdash\!\dashv$	
prime 7 effect	0	$\vdash$	$\vdash$
prime 11 effect	9	$\bullet$	$\vdash$
prime 13 effect	1	$\bullet$	$\vdash$
prime 17 effect	11	$\vdash$	$\vdash$
prime 19 effect	12	$\dashv$	-
prime 23 effect	0	$\bullet$	-

T_{27} (805) also finishes up as a quadruplet: 821, 823, 827, and 829 .

The TNumbers 1 to 27, inclusive, are special cases in this study, they do not fit into the general scheme that is yet to be described because of how the primes "layer" themselves under the TNumbers up until T_{28}.

In the diagram above the area denoted by the thick blue line indicates the start of prime 29's influence at T_{28}. Every Tnumber after this will have to be analyzed by each of the primes 7 through 29, or at least up to the point where the Tuplet becomes destroyed. More importantly, however, is that we now have a rectangle beginning at T_{28} and extending to infinity. This is of use as we see in the next section.

5.3.11 Prime Basis

The chaos and general sextuplet destructiveness of the primes 7-29 is simply a given. The power of 7 and 11 alone to destroy sextuplets is gargantuan. Out of an infinity of choices where should we look for sextuplet survivors?

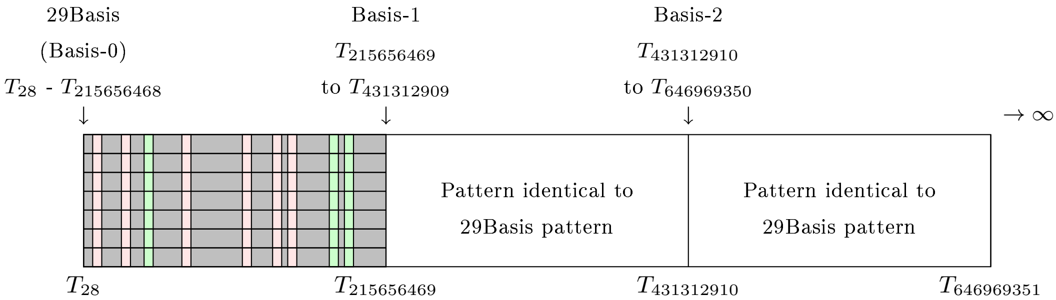
We will use the 29Basis to help us. The 29Basis, the only useful pattern for primes 7-29, is a catalog of the only locations, by TNumber, where sextuplets, and/or other Tuplets, are possible. The 29Basis is functional out to infinity because it is re-usable.

The 29Basis catalog is produced by simply calculating, in order, *all the combinations* of prime 7 through 29 using the method outlined in section Usage of primes 7-29 on page 19.

This is not as bad as it sounds! The total combinations to calculate will be 7-29 primorial; i.e., $7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 29 = 215656441$.

We only need to iterate over 215,656,441 TNumbers. This is sufficient since the pattern, and results, will then repeat. This means the 29Basis is constructed over T_{28} to $T_{215656468}$, inclusive. When we reach $T_{215656468}$ we can stop because the pattern repeats beginning with $T_{215656469}$.

In the diagram below the 29Basis, as well as its re-usable nature, is sketched out. In it we've performed the crossing number effect for each TNumber and green areas represent sextuplets which have survived, pink other surviving Tuplets, and gray are destroyed 'tuplets. That 29basis then repeats forever.



We can now throw out the grays and keep the greens. Optionally, if one is also interested in quintuplets and quadruplets, keep the pinks. The calculation process only needs to be done once, and it takes a computer only a minute or two to calculate and write to a file (file size about 190Mb). The resulting file is the 29Basis: a catalog of the only TNumbers where Tuplets are possible (the program I wrote will make this file for you, if you prefer not to code something yourself).

Once one has this catalog there is no further need to consider primes less than 31, the primes 7-29 have done their job and provided a usable pattern for us to use and made the discovery and study of sextuplets viable.

A completed 29Basis file has the following statistics. Remember that these 29Basis locations are only where Tuplets *can* be, it will take the primes 31-59 to make the final decision if the Tuplet will survive:

1,956,955 Sextuplets
5,010,341 Left sided Quintuplets
5,010,341 Right sided Quintuplets
5,528,488 Quadruplets
~17.5 million TNumbers out of the 215 million+ that we searched

The following constants define the limits of bases: Basis begin (B), Basis end (E), and Basis length (L). Remember that the begin and end are inclusive:
$B_B = 28$
$B_E = 215656468$
$B_L = 215656441$

The 29Basis starts at T_{28}, which begins with integer 835, and extends to and includes $T_{215656468}$, which ends with integer 6,469,694,064, a range which covers many needs. And it is very easy to use the 29Basis to find and study any number range we like. The catalog is infinitely re-usable because one only needs to add 215,656,441 to any TNumber from the 29basis file to to move to the next basis, Basis-1 which has the range as shown in the figure above. If you want to study numbers in Basis-2 then add $(215,656,441 \times 2)$ to the 29basis TNumber. Or, more generally, use the following relation where T_{bc} is a TNumber from the 29basis catalog and b is the basis number ($b \geq 0$):

$$T_{\text{bc}} = T_{\text{bc}} + B_L b \tag{7}$$

Basis-0, then, is the 29Basis file TNumbers unaltered since $b = 0$
Basis-1, $b = 1$, will analyze the 29basis TNumbers as being in the $T_{215656469}$ to $T_{431312909}$ range, and so on.

Following are the relationships that connect bases and TNumbers (if you want to use regular number line positive integers then convert them to their TNumbers first).

Find the TNumber range, beginning (T_b) and ending (T_e), for any Basis number b:

$$T_b = B_B + B_L b \tag{8}$$
$$T_e = B_E + B_L b \tag{9}$$

Given any TNumber, T_n, ($n \geq B_B$) return the Basis number, b, it resides in:

$$b = (T_n - B_B)\,\text{div} B_L \tag{10}$$

One can, of course, search any range of numbers in any way one likes: by TNumber, by Basis, or by range of integers greater than 835. But the advantage of Basis based TNumber analysis is completeness, nothing can be missed, all numbers will be guaranteed to have been tested. If one tested by groups of 1 million, for instance, it is quite possible that some findings will be duplicated because of the peculiarities of integer to TNumber conversion. Or, it could be that a entire range was missed as it becomes difficult to accurately type large integers by hand. For these reasons, I recommend using Basis based analysis.

In summary so far: we've used the primes 2,3,5 and the primes 7-29 to build natural groupings in such a manner as to obviate the need to use primes 2-29. There is no need to consider any prime less than 31 for the remainder of this study.

5.3.12 Twin Sextuplets

Given that the closest two Sextuplets can be to each other, by TNumber, is 7 (as discussed in section 5.3.2 on page 11), then it follows that a "twin sextuplet" must be of the form T_n and T_{n+7}.

How likely are they? Analysis of my 29Basis file shows there are 106,029 *possible* twin sextuplets. So far I've checked for twin sextuplets through Basis-40 and have found none, and so, if twin sextuplets exist in actuality, they must be in the range greater than $T_{8841914108}$, or greater than integer 265,257,423,264. This is an ongoing study as I have not found references to twin sextuplets on the internet.

Twin sextuplets will have the relationships as below, where T is any TNumber and $T_2 > T_1$, and N is any corresponding prime member of separate sextuplets and $N_2 > N_1$:

$$\frac{T_2 - T_1}{7} = 1 \text{ or } T_2 - T_1 = 7 \tag{11}$$

$$\frac{N_2 - N_1}{7} = 30 \text{ or } N_2 - N_1 = 210 \tag{12}$$

5.4 Tuplet Math

For a review of symbol meanings see section Symbols for template diagrams and prime structures on page 9.

Each of our TNumbers starts out as a pristine Sextuplet. As time goes by and other primes cross into that TNumber they have effects which are added to what has come previously. The result can either be survival, change, or destruction. The tables below show a selection of symbol addition in detail, the idea is easy to see. The numbers in the header are the 0-based indices of the sextuplet positions in a Prime Template. The Symbol X denotes that the effect of a prime number crossing into that TNumber was to strike, i.e., remove, that position.

Four basic examples are shown below in table form. Once an X is inherited it stays with the TNumber. An X is when the prime strikes either 16, 18, 22, or 24 Prime Template index:

	Symbol	12				16		18				22		24				28
Sextuplet	├┤	o				o		o				o		o				o
+ Sextuplet	├┤	o				o		o				o		o				o
=	├┤	o				o		o				o		o				o

	Symbol	12				16		18				22		24				28
Sextuplet	├┤	o				o		o				o		o				o
+ {16|18|22|24}		o				X		o				o		o				o
=	destroyed	o				X		o				o		o				o

	Symbol	12				16		18				22		24				28
Sextuplet	├┤	o				o		o				o		o				o
+ R. Quintuplet	⊣	X				o		o				o		o				o
=	⊣	X				o		o				o		o				o

	Symbol	12				16		18				22		24				28
Sextuplet	├┤	o				o		o				o		o				o
+ L. Quintuplet	⊢	o				o		o				o		o				X
=	⊢	o				o		o				o		o				X
+ R. Quintuplet	⊣	X				o		o				o		o				o
=	-	X				o		o				o		o				X

5.4.1 Summary of symbol addition:

A note about the quadruplet: They do not occur in nature. There is no crossing of any prime that produces a quadruplet effect. Crossing effects can only be 1) allowing a sextuplet to survive, 2) applying a right quintuplet effect, or 3) applying a left quintuplet effect. The "sum" can never return to a previous state when it is changed, it can only remain in the current state, be changed to another state in accorance with the rules below, or destroyed.

Quadruplets are the marriage of a right quintuplet and a left quintuplet effect, those natural quadruplets are starred green. While the quadruplet is represented in the tables for completeness, it is not always relevant in that no crossing effect can ever be a quadruplet, those cases are starred red. If a TNumber sum result reaches a quadruplet state nothing can change that state unless it is destroyed by a {16|18|22|24=X} strike.

The computer codes in current use that represent the effects are also shown.

current state		+ effect		=	
⊢⊣	0	⊢⊣	0	⊢⊣	0
⊢⊣	0	⊢	1	⊢	1
⊢⊣	0	⊣	2	⊣	2
⊢⊣	0	-★	3	-	3
⊢⊣	0	X	4	X	4
...		...		...	
⊢	1	⊢⊣	0	⊢	1
⊢	1	⊢	1	⊢	1
⊢	1	⊣	2	-★	3
⊢	1	-★	3	-	3
⊢	1	X	4	X	4
...		...		...	
⊣	2	⊢⊣	0	⊣	2
⊣	2	⊢	1	-★	3
⊣	2	⊣	2	⊣	2
⊣	2	-★	3	-	3
⊣	2	X	4	X	4
...		...		...	
-	3	{any}		-	3
X	4	{any}		X	4

5.5 Primes 31 through 59

The primes 31-59 are the deciders of the ultimate fate of any 29Basis Template's effect. These primes form a group that will represent all primes, but that are not necessarily prime at every level, and so it is better to call them now, what they really are: *potential primes*. To signify that they will either be noted as "potential prime", "potPrime", Q, or pP().

The potential prime families are: 31, 37, 41, 43, 47, 49, 53, 59

Those values are the base values of that potPrime family; i.e., $Q^{31}_{\text{base}} = 31$, $Q^{37}_{\text{base}} = 37$, and so on. They are the first "primes" of the form where their values mod 30 are: 1, 7, 11, 13, 17, 19, 23, and 29

When there is any need to identify a particular member of a potPrime family it will be denoted as Q^{31} or pP(31), Q^{37} or pP(37), and so on.

What follows will be detailed descriptions, and derivations, of the structures, patterns, and relationships of the potPrimes. If you just want to know how to use them for sextuplet generation then see Usage of primes 31-59 on page 39

5.5.1 Potential Prime properties

The potPrimes are very regular, follow regular patterns, and are amenable to equations and structures. They can represent all primes ≥ 31 simply by adding products of 30 to each. For instance, 31, 61, 91... mod 30 = 1. Similarly, 49, 79, 109... mod 30 = 19, and so on.

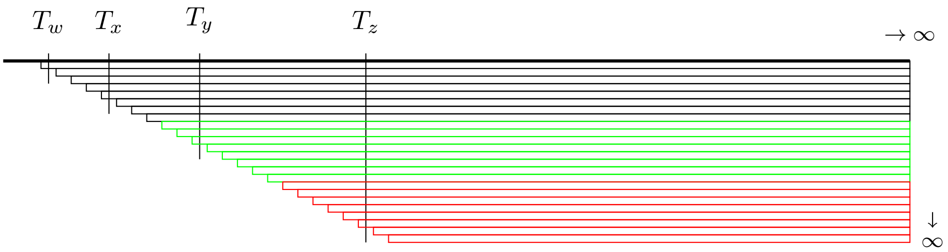
The relation below shows how any prime ≥ 31 can be constructed, where C is our constant 30, $n \geq 0$, and Q_{base} is the potPrime's base value :

$$Q_n = Q_{\text{base}} + Cn \tag{13}$$

Examples: $Q_0^{37} = 37$, $Q_2^{49} = 109$, and so on.

Their natural progressions through Prime Templates are developed in the same fashion as they were for the primes 7-29 (5.3.1 on page 10).

The following diagram illustrates the structure we are building with these primes. It is diagrammatic and does not show the exponential nature of squares of succeeding primes:



The thick black line is the number line, in TNumbers, that, if coming from the 29basis, are already pre-processed by the primes 7-29.

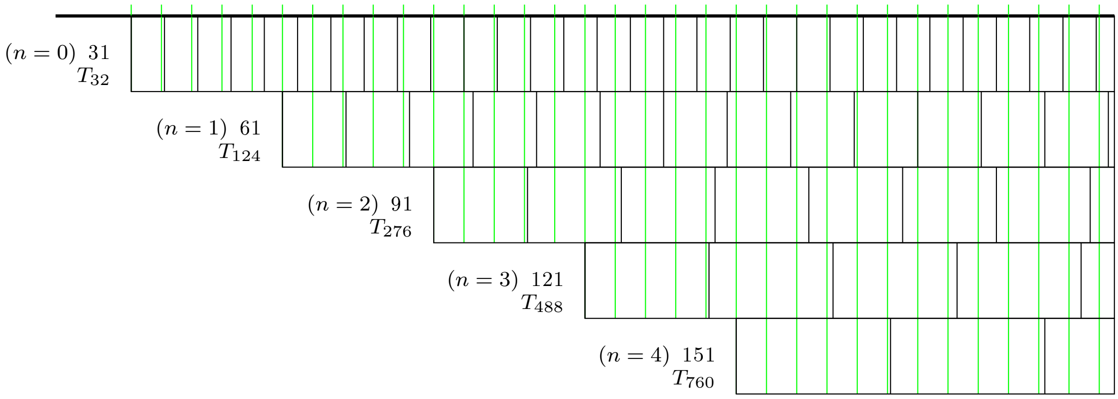
The beginning of each rectangle represents the location where a potential prime becomes effective; that is, at its base value squared. The color groups correspond to the n in $Q_n = Q_0 + Cn$ (equation 13), so black is $n = 0$, green $n = 1$, red $n = 2$, and so on to infinity.

The 8 rectangles of the black group are potPrimes 31, 37, 41,..., the green group to the 8 potPrimes of 61, 67, 71,..., and the red group to 91, 97, 101,...

We want to evaluate TNumbers coming from "above", and to do this we must iterate through any potPrime that is "effective" at that point. For T_w it is potPrime 31 (Q_0^{31}). For T_x it requires processing by potPrimes 31...47 before we can be certain we've confirmed its sextuplet-ness. For T_y it is the potPrimes 31...71, and for T_z 31...109.

To visualize the relationships within a family the following diagram focuses on Q^{31}. The first 3 rows correspond to the first black row, first green row, and first red row from the diagram above. This diagram is also not to scale or exponentially accurate.

The green lines are evenly spaced represenions of 30 TNumbers, the ever expanding black line regions represent the nature of the increasing size of potPrimes at that n. This shows the ever changing relationship between Q_n and C, in addition to the relationships between members of the same family against each other. It also shows quite clearly the nature of skipping Templates by the potPrimes, discussed later. While this is a nice way to visualize what is happening as potPrimes move through the TNumbers, the green/black region's relative lengths are approximated for demonstration purposes and not at all accurate or in scale.



Our goal is translate this entire structure into equations that can give confirmation of sextuplet survival, or destruction, at any TNumber we like, and that such equations can help us investigate sexuplet discovery, and, perhaps, hints as to whether sextuplets are infinite.

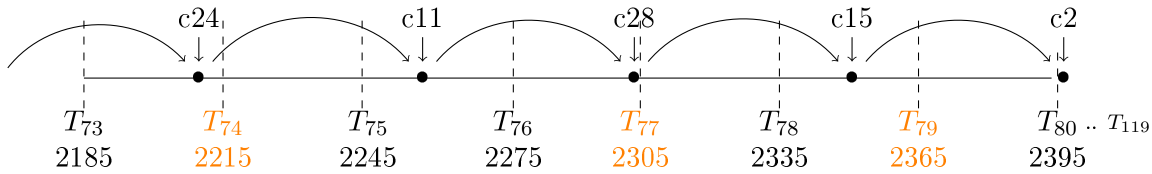
5.5.2 Defining properties of potential primes

STRAIGHTFORWARD CROSSING NUMBERS:

One big advantage is that the point that they strike when crossing is exactly that point; for example, if the crossing number is 12 then it has struck position 12 exactly. This is because all these primes are greater than the Template length: They can hit only one point in a Template and not multiple points like the primes 7-29. Therefore the crossing number and positional strike are equal. It also means that a crossing number of, for example, 12, the right handed quintuplet position, is the same for all potPrimes.

SKIPPING PRIME TEMPLATES:

In addition, again since each is > 30, there must be at least one or more TNumbers that are completely skipped for each potPrime. In fact, the number of skippings over each potPrime's pattern are, respectively, 1, 7, 11, 13, 17, 19, 23 and 29. The figure below for pP(47) illustrates this, pP(47)'s effective TNumber (remember effective TNumber is based on the prime's square which for 47 is 2209) is T_{73} (2185). The arcs encompass ranges of 47, while the distance between TNumbers is, of course, 30. The figure is diagrammatic, not to scale, and not complete (the last TNumber in 47's natural progression is T_{119}):



One sees how T_{74}, T_{77}, and T_{79} are skipped. In the summary tables of the potPrimes Details such skips are symbolized by "o".

INFLATION:

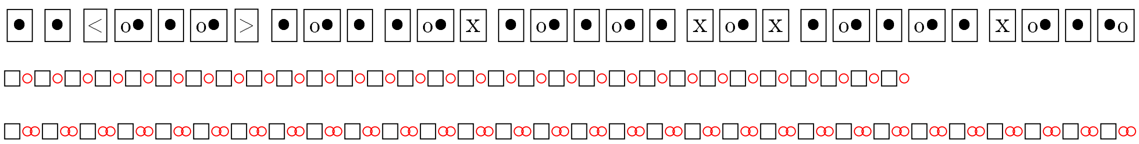
These 8 potPrime families suffice for all calculations involving finding effects at any TNumber we choose because the natural progressions for each member of the potPrime family is identical to the base value's natural progression (with one caveat). For example, the natural progression of pP(61) is the same as it is for pP(31). The natural progression of pP(91) is identical to pP(61) is identical to pP(31). This is true for every Q_n in family Q.

The caveat is that pP(61) is bigger than pP(31) by 30. It turns out, however, that this is compensated for by inserting a skip before every crossing in the natural progression; that is, the succeeding potPrime members in a family inflate the natural progression by a constant, smooth amount.

Every crossing for a potPrime gets an extra skip for every $n > 0$. For example, $Q^{31}_{n=1}$ (equal to 61) will have one skip added for each of the crossings of the natural progression, $Q^{31}_{n=2}$ (equal to 91), will have 2 skips added, and so on.

These smooth inflations are proven by mapping out pP(61)'s (and/or ...pP(91)'s, and/or pP(121)'s,...) natural progressions in the same fashion as we have used for all primes. If one makes this effort in mappings then one will see the identical, smooth expansion as described. It is analogous to the expansion of the Universe: the expansion takes place at every location.

In the diagram below we look at the Q^{41} natural progression diagramatically and show how expansion works. There are 30 boxes showing the Prime Template strikes 41 makes, the indexing of these boxes is 0...29. An "o" inside a box indicates that 41 has skipped a Template before arriving in that box. The "<" stands for a left handed quintuplet strike, the ">" a right handed quintuplet strike, and "x" for a destroying strike. The black dot means it has no effect:



In the second and third rows, for space considerations, the boxes from the first row are summarized by blank boxes, but they contain exactly the same information one sees in row 1. Row 1 is $n = 0$ (i.e., Q^{41}_0). Row 2 shows the natural progression of Q^{41} at $n = 1$, that is the potential prime 71. It leaps over an extra template at each jump because it is

30 bigger than 41. Row 3 is $n = 2$, potPrime 101, and it must leap over 2 templates each time. The contents of the boxes do not change, and so one sees the natural progressions do not change among members of a potPrime family, they just become inflated.

The diagram, row 1, also shows how the two cycles of 30 and 41 combined interact. The cycle can be looked in two ways: There are 41 30-cycles, or there are 30 41-cycles, before the pattern repeats. The potPrime 41 strikes exactly 30 Templates in the combined cycle because it skips some Templates. The 30 41's are the manner in which 41 soars from one Template to strike into the next (sometimes skipping one). I've already mentioned there are 30 boxes in the diagram. If you now count the symbols inside the boxes the total will be 41. This type of interaction cycle is the same for all Q.

By the way, an inflation space is never put before the first template crossing (position 0). That always lines up correctly because the potPrimes are interval-ized (discussed soon). When doing natural progressions by hand, it may appear that one needs to insert a skip space before the first template crossing, but this is illusion. Those phantom skip spaces can be added to the end of the sequence instead with no loss of meaning or functionality.

This inflation for pP(41)'s family is also true for every potPrime (pP(31),pP(37), pP(43),...) which means that a single equation can be derived that gives the relevant3 effective starting TNumber for any potPrime we like, as far out as we like. That derivation follows below. It also means that the inflation can be summarized in tables, this is discussed next.

CROSSING EFFECT LOOKUPS:

Since we now have a family of primes describing crossing number effects out to infinity, a simple direct look up relationship, such as we had for primes 7-29, no longer suffices. This is particularly true since we want to use unifying equations and we know potPrime members natural progressions inflate making crossings a moving target.

Effect lookup is replaced by lookup tables that are derived from each respective potPrime's natural progression. It consists of a crossing offset and a multiplication factor which will then adjust any expanded potPrime (i.e., $Q_{n \geq 1}$) to give back its effect at any TNumber position in the number line. These lookup tables can be considered "constants" of each respective potPrime.

We will re-create the lookup table for Q^{41}. Natural progressions are, by definition, 30 positions long and map out the effects a potPrime has as it travels though its cycle striking into Prime Templates. The natural progressions are labeled in 0-based notation so the indices of the positions will be $0 \ldots 29$ and $0 \ldots 40$. In the following figure the numbers above are the cumulative sum of symbols in the boxes at that box, the numbers below are the index of that box:

c	0	1	2	4	5	7	8	9	11	12	13	15	16	17	19	20	22	23	24	26	27	28	30	31	33	34	35	37	38	40
	●	●	<	o●	●	o●	>	●	o●	●	●	o●	X	●	o●	●	o●	●	X	o●	X	●	o●	●	o●	●	X	o●	●	●o
q	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29

We only need to test 6 positions for any potPrime, the 4 destructors (x) and the 2 quintuplet strikes (“$<$” and “$>$”). I have traditionally used c for the crossing effect, and q for the multiplier in my tables.

The crossing effect, c, is *as it relates to* the potential prime. This means c mirrors the 41 cycle and so is constructed by counting, starting with 0, the symbols within the boxes, and noting that number at each of the symbols of interest. This puts “$<$” at 2, “$>$” at 8, and the x’s at 16, 24, 27, and 35. These positions are where the effect is actualized by the natural progression cycle of 41 striking sextuplet positions in the prime templates.

The multiplier, q, is the natural progression as it relates to Template length and so is mirrored by the boxes. This means q takes on the values of the natural progression index and so can be read off directly from the natural progression indices. This puts “$<$” at 2, “$>$” at 6, and the 4 x’s at 12, 18, 20, and 26. These are the positions of the boxes, and we know each box will have n inflation spaces added in between. Therefore, these q describe the inflation of potPrimes at any natural progression index.

c	q	effect
2	2	$<$
8	6	$>$
16	12	X
24	18	X
27	20	X
35	26	X

This lookup table means we never have to expand any Q, or in any way worry about how to find an effect for any $Q_{n\geq 0}$ at any TNumber. All we need is n, the same n we use in the relationship $Q_n = Q_o + Cn$.

5.5.3 Using Potential Prime Effect Lookup Tables

As an example we will use the table developed immediately above this section.

The following equation governs the usage of the lookup table, where i is the calculated offset, $n \geq 0$, and c, q are iterated from the table rows:

$$i = c + qn \tag{14}$$

The variable i is then used to compare to the relative offset, R, of any target TNumber we are testing (discussed in section 5.5.7 on page 37):

If $i < R$ continue on to the next row and test again.

If $i > R$ stop testing, the index is beyond the relative position of the TNumber we are testing.

If $i = R$ stop testing, get the effect from the table and apply symbol math (5.4 on page 23) to the cumulative effect result.

The $i = R$ condition is also where to check whether testing will continue on to the next potPrime for the target TNumber. If we are interested only in sextuplets then any $i = R$ stops the testing for that target, the result can no longer be a sextuplet. If, however, we are interested in the other Tuplets as well, then only when the effect is “X” do we not continue to the next potPrime.

The variable n is becoming indispensable to the net of relationships within a potPrime family. We will be deriving, shortly, the equation for getting n after we derive an equation for getting a potPrime effective TNumber. Following that will be a discussion of finding the relative offset of any TNumber.

It should be noted that results of these tests are/will be made in TNumbers. So we need to know how to convert any successful results into the actual integers of the prime tuplet, that is described next.

5.5.4 Convert successful result into Tuplet members

For an overview of the process of testing TNumbers see Usage of primes 31-59 on page 39.

If a target TNumber passes all tests then it contains a Tuplet. Generally, if one is generating lists of Tuplets, one wants to see the actual Tuplet members in integer form.

To convert successful results into the Tuplet members first convert the target TNumber, T_T, to its integer location I: $I = (T_T \times 30) - 5$ (eq. 4 on page 9).

Each member, $M_{1...6}$, always has the same offset inside a Prime Template (see fig. 4, section 5.1 on page 8), so the process is simply to add that constant value to the TNumber's integer value:

$$\begin{aligned} M_1 &= I + 12 \\ M_2 &= I + 16 \\ M_3 &= I + 18 \\ M_4 &= I + 22 \\ M_5 &= I + 24 \\ M_6 &= I + 28 \end{aligned} \tag{15}$$

If the Tuplet is a left sided quintuplet then the 28 offset is omitted, if a right sided quintuplet the 12 is ommited, and both of these are omitted for quadruplets.

Example: T_{647} contains a sextuplet. $(647 \times 30) - 5 = 19405$, and so the sextuplet members are:

$$19417 = 19405 + 12$$
$$19421 = 19405 + 16$$
$$19423 = 19405 + 18$$
$$19427 = 19405 + 22$$
$$19429 = 19405 + 24$$
$$19433 = 19405 + 28$$

5.5.5 Derivation for getting Potential Prime effective TNumber

For this derivation we will use Q^{31} as the example but the results are the same for all potPrime families. We will use p to represent the pP()'s base value ($Q^{31}_{\text{base}} = Q^{31}_0 = 31$), $C = 30$ (the length of the Prime Template), and $n \geq 0$.

In order to use potPrimes to discover Tuplets we need to know at which TNumber each family's members become effective (see initial assertions, section 4.1 on page 4). If the potPrime member is effective it means we must process that potPrime to see how it effects the Sextuplet we are testing.

We've seen (equation 13 on page 26) that members are calculated by taking the pP() base value and adding 30 as many times as we like. The relationship is simple: $p_n = p + Cn$. For example, where $n = 0, 1, 2, 3$ we get 31, 61, 91, and 121.

In order to align all these results properly to TNumbers we will need the interval concept from the initial assertions discussed in section 4.2 on page 5, from which we will use the final general form of the squares interval equation (equation 2 on page 6): $x = 2p+C+2Cn$, and we will set its initial value to p and the interval to C. This will give us x, the number of intervals of size C between successive members of the potPrime family squared if we modify it to use the previous n: $x = 2p + C + 2C\,(n-1)$. We can see that the portion $2p+C$ will be constant for each potPrime, for $p = 31$ it is 92. And, since $2C$ is also constant equaling 60, we can for now write the equation as: $x = 92 + 60\,(n-1)$.

From this we can expect that the difference between the TNumbers where the squares of form $(p + Cn)^2$ reside will be equal to $92 + 60\,(n-1)$:

n	$p+Cn$	$(p+Cn)^2$	T	T_{current}-T_{prior}		$92 + 60\,(n-1)$
0	31	961	32	n/a, start of series		n/a, n must be ≥ 0
1	61	3721	124	$124 - 32 = 92$		92
2	91	8281	276	$276 - 124 = 152$		152
3	121	14641	488	$488 - 276 = 212$		212

Indeed, the difference between TNumbers holding the values of successive squares of potPrimes is described by the equation, and the differences from n to n are 60 greater ($2C$) each time. The potPrimes are now interval-ized and the extent of the regions of influence will always be an integer multiple of our interval value; that is, they are now interval-ized for TNumbers.

For example, there are exactly 92 intervals of size 30 (TNumbers) between T_{32} and T_{124}. The difference between TNumbers which contain the values of 31^2 and 61^2 is exactly 152 Templates, and so on.

We are now able to write an equation allowing us to calculate the effective TNumber between succeeding levels of n by using the previous effective TNumber. The first TNumber is the beginning of the series, which is the potPrimes starting TNumber generally symbolized by Q_{start}. For Q^{31}_{start} the value is $T_{32} = 32$. By the way, all of the constants associated with each potPrime are shown in each potPrime's "Details sections" which follow a few sections below.

n	$T_{\text{prevous}} + 92 + 60\,(n-1)$	predicted T Number
0	given, constant	$T_{32} = 32$
1	$32 + 92 + 60\,(0)$	124
2	$124 + 92 + 60\,(1)$	276
3	$276 + 92 + 60\,(2)$	488

These do indeed match the calculated TNumbers in the previous table. We can now develop the fully generalized equation. To do so we will use a special form of equation so we can watch a wonderful pattern develop, T_{eff} is the TNumber at which the potPrime becomes effective for any n:

$$T_{\text{eff}} = Q_{\text{start}} + n\,(2p + C) + \left(\sum_{n=1}^{n} n\,(2C)\right) - n\,(2C) \tag{16}$$

We need to sum the $+n(2C)$ term because those results are cumulative. We will stop using the literal 60 and return to $2C$, but will replace the constant $2p + C$ with 92:

$$T_{\text{eff}} = 32 + n\,(92) + \left(\sum_{n=1}^{n} n\,(2C)\right) - n\,(2C)$$

The $-n\,(2C)$ term adjusts for the effect of looking back one result, the hack of T_{prevous} we used in the last table we constructed. Here are the results of our equation for up to $n = 5$:

n	$T_{\text{eff}} = 32 + n\,(92) + \sum_0^n (n\,(2C)) - n\,(2C)$	T_{eff}
0	$32 + 0\,(92) + 0\,(2C) - 0\,(2C)$	32
1	$32 + 1\,(92) + 1\,(2C) - 1\,(2C)$	124
2	$32 + 2\,(92) + 2\,(2C) + 1\,(2C) - 2\,(2C)$	276
3	$32 + 3\,(92) + 3\,(2C) + 2\,(2C) + 1\,(2C) - 3\,(2C)$	488
4	$32 + 4\,(92) + 4\,(2C) + 3\,(2C) + 2\,(2C) + 1\,(2C) - 4\,(2C)$	760
5	$32 + 5\,(92) + 5\,(2C) + 4\,(2C) + 3\,(2C) + 2\,(2C) + 1\,(2C) - 5\,(2C)$	1092

One sees that the summed term becomes the same problem as Carl Friedrich Gauss' (in)famous add the numbers from 1 to 100 school boy tale. The general form of Gauss' solution is, where the sought solution runs from 1 to n: $\frac{n(n+1)}{2}$. For Gauss' problem the answer is $\frac{100(101)}{2} = \frac{10100}{2} = 5050$.

For our problem, $5\,(2C)+4\,(2C)+3\,(2C)+2\,(2C)+1\,(2C)$, we factor out $2C$: $(5+4+3+2+1)\,(2C)$, and then use the Gauss formula, $\left(\frac{5(6)}{2}\right)2C$. or more generally $\left(\frac{n(n+1)}{2}\right)2C = \left(\frac{n^2+n}{2}\right)2C$.

Alternatively, the ending negative term can be removed by canceling the first term of the sum in which case we would use $n-1$ instead of n: $\left(\frac{n-1(n-1+1)}{2}\right)2C = \left(\frac{n^2-n}{2}\right)2C$ in which case the negative term is left out of the final equation.

Putting our results back into equation 16 on the preceding page using symbols. Both the n and $n-1$ form are displayed:

$$T_{\text{eff}} = Q_{\text{start}} + 2Q_{\text{base}}n + Cn + \left(\frac{n^2+n}{2}\right)2C - 2Cn$$
$$T_{\text{eff}} = Q_{\text{start}} + 2Q_{\text{base}}n + Cn + \left(\frac{n^2-n}{2}\right)2C$$

Simplifying to the final form:

$$T_{\text{eff}} = Q_{\text{start}} + 2Q_{\text{base}}n + Cn^2 \tag{17}$$

Putting it in terms of pP(31) which we've used for the examples:

$$T_{\text{eff}} = Q^{31}_{\text{start}} + 2Q^{31}_{\text{base}}n + Cn^2$$
$$T_{\text{eff}} = 32 + 2\,(31)\,n + Cn^2$$
$$T_{\text{eff}} = 32 + 62n + Cn^2$$

We now have a general equation to find the effective TNumber for any family member of any of the potPrimes at any location in the number line. Given T_{T} is the target TNumber you are testing, n is the level currently being evaluated, and T_n is the T_{eff} at that n, then when $T_{\text{n}} > T_T$ you know that potPrime member, Q_n, and greater, $Q_{>n}$, can be ignored, they will not affect the outcome of your sextuplet testing.

5.5.6 Getting "n"

Rather than guessing at each TNumber we want to test by testing values of n until the effective TNumber exceeds our target TNumber, it would be preferable to know n ahead of time so we know exactly how many potPrimes to test.

We can in fact do this by solving for n in equation 17 on the previous page. We will, however, change it slightly:

$$T_T = Q_{\text{start}} + Cn^2 + 2Q_{\text{base}}n$$

The T_{eff} of the original equation is now T_T which represents the target TNumber; i.e., the TNumber we want to test. We will ultimately need to floor the result so the returned integer will be to the nearest n that will produce *only* effective TNumbers.

There are two forms for the derivation. The first requires the potPrime details of its starting TNumber and its base value. This results in a complex formula. We will also derive a simpler version which requires the potPrime details of its base value and a calculated adjustment. Both formulas, if calculated to the appropriate precision, give the same result.

Let p stand for Q_{base}, T_T must be ≥ 32 (before T_{32} no potPrime *can* be in effect). We will need to complete the square at one point:

$$\begin{aligned}
T_T &= Q_{\text{start}} + Cn^2 + 2pn \\
T_T - Q_{\text{start}} &= Cn^2 + 2pn \\
T_T - Q_{\text{start}} + \frac{p^2}{C} &= Cn^2 + 2pn + \frac{p^2}{C} \\
T_T - Q_{\text{start}} + \frac{p^2}{C} &= \left(\sqrt{C}n + \frac{p}{\sqrt{C}}\right)^2 \\
\sqrt{T_T - Q_{\text{start}} + \frac{p^2}{C}} &= \sqrt{C}n + \frac{p}{\sqrt{C}} \\
\left\lfloor \frac{\sqrt{T_T - Q_{\text{start}} + \frac{p^2}{C}} - \frac{p}{\sqrt{C}}}{\sqrt{C}} \right\rfloor &= n
\end{aligned}$$

Replacing with standard symbols:

$$n = \left\lfloor \frac{\sqrt{T_T - Q_{\text{start}} + \frac{(Q_{\text{base}})^2}{C}} - \frac{Q_{\text{base}}}{\sqrt{C}}}{\sqrt{C}} \right\rfloor \tag{18}$$

If the result, before flooring is negative, then the pot prime is not active yet and that potPrime can be ignored. For example, Q^{37} becomes effective at T_{45}, if you try to find n for Q^{37} in the range of T_{32} to T_{44} the equation result is, before flooring, negative, and Q^{37} will have no effect at that T_T.

The complexity of equation (18) stems from, it turns out, the fact that Q_{start} is not in terms of p and C. This can be fixed, Q_{start} is calculated from equation 3 on page 8: $\left\lfloor \frac{N+5}{30} \right\rfloor$ where N is $(Q_{\text{base}})^2$. Again letting p stand for Q_{base} this becomes $\left\lfloor \frac{p^2+5}{30} \right\rfloor$. Now we have it in terms of p and C, but since it is floored it will fail to give a proper result in edge cases.

If we could adjust this term to always give an integer result when divided by 30 we would have what we need. And this is only possible if we add a new piece of information. The squares of potPrime base values always either end with digit 1 or digit 9. Another way of saying this is $p^2 \mod 30 = 1$ or 19. If we adjust the expression properly the division by 30 will always be an integer value and the floor function can be removed.

For potPrimes $= 1 \mod 30$ we can use $\frac{p^2-1}{30}$ and for potPrimes $= 19 \mod 30$ we can use $\frac{p^2-19}{30}$, no need to floor the results, these divisions are exact integers. This means we can replace Q_{start} with $\frac{p^2-a}{30}$, where a is the adjustment factor. For this reason each potPrime has a property Q_{adj} which is either 1 or 19, depending.

Again let p stand for Q_{base} and using our generalized Q_{start}. The final form must be floored:

$$\begin{aligned}
T_T &= \frac{p^2-a}{C} + Cn^2 + 2pn \\
T_T C &= p^2 - a + C^2n^2 + 2pCn \\
T_T C + a &= p^2 + C^2n^2 + 2pCn \\
T_T C + a &= (p + Cn)^2 \\
\sqrt{T_T C + a} &= p + Cn \\
\left\lfloor \frac{\sqrt{T_T C + a} - p}{C} \right\rfloor &= n
\end{aligned}$$

Replacing with standard symbols, Q_{adj} must be either 1 or 19 which comes from the result of $(Q_{\text{base}})^2 \mod 30$:

$$n = \left\lfloor \frac{\sqrt{T_T C + Q_{\text{adj}}} - Q_{\text{base}}}{C} \right\rfloor \tag{19}$$

From equation 19 or 18 on the preceding page we can now get the appropriate n at any TNumber which we can use to generate the appropriate number of potPrimes that we will need in order to thoroughly test a TNumber for Sextuplet survival, see Usage of primes 31-59 5.5.8 on page 39.

It could be argued that there is no need to present two equations for the getting of n. However n is central to these structures and must be exact. I have found that it was very helpful to have two equations for testing, comparing results, and getting sanity checks. Perhaps you will find them helpful also.

5.5.7 Getting relative offset for any TNumber

You have a TNumber that you would like to check to see if it has a sextuplet in it or not. To do that you will need to test it against the potPrime's Effect Lookup Tables for all n (5.5.3 on page 30). That testing method requires that the target TNumber's relative offset at that n is sent in to compare the Lookup test against.

NOTE: As of the middle of March 2021, another method was developed that uses regular arithmetic mod rather than the special TNumber mod method below. And yet another even simpler method was found in the middle of April 2021. The newer equations are faster for computer programs. The derived from first principles TNumber mod method is shown below, the newer methods will be discussed after. The third method is recommended for its simplicity, all three equations give the same results.

Since the Lookup test returns its compare index as an index into the (often expanded) potPrime's natural progression, we need to map the target TNumber as a relative offset into the same (often expanded) potPrime natural progression.

Basically, this boils down to computing the number of complete cycles, in TNumbers, of that potPrime that are less than the target TNumber. Subtract that from the target TNumber to get back how far inside that last cycle the target is; that is, its relative offset to the pristine potPrime's natural progression. In other words, we need to normalize the target TNumber to the potPrime's natural progression at any n.

The following equation does that. The result, R, is the relative offset that the Lookup test requires to compare against, T_T is the target TNumber, $\mathsf{T}(Q,n)$ is the result of the effective TNumber equation (17 on page 34) for the Q at that n, $n \geq 0$, $T_T \geq 32$, and $T_T \geq \mathsf{T}(Q,n)$:

$$R = T_T - \left(\left\lfloor \frac{T_T - \mathsf{T}(Q,n)}{Q_{\text{base}} + Cn} \right\rfloor (Q_{\text{base}} + Cn) + \mathsf{T}(Q,n) \right) \tag{20}$$

13.03.2021: A method using a simpler equation that does not require an effective TNumber has also been developed using a regular arithmetic modulo function. When used in a computer program it produces an about 30% speed increase over the equation above.

The "mod" function has no knowledge of TNumbers and their start at integer 25 and so, in order to use a regular mod result, the target TNumber must be "unrolled" so that the resulting R is identical to the one obtained from the equation above.

In essence: Given a target TNumber, T_T, and taking the mod result by Q_n, one must remove the extra space of expansion (which is dependent on n), and also an adjustment amount that is constant for each potPrime family which relates to the offset and "compression" of the TNumber system. The constant adjustment amount is calculated as follows :

$$k = C + Q_{\text{start}} \mod Q_{\text{base}} - Q_{\text{base}} \tag{21}$$

Calculating k for Q^{53} for example, $C = 30$, Q^{53}_{start} =93, $Q^{53}_{\text{base}} = 53$, $Q^{53}_{\text{base}} \mod 30 = 23$:

$$\begin{aligned} Q^{53}_k &= 30 + 93 \mod 53 - 53 \\ 17 &= 30 + 40 - 53 \text{ or} \\ 17 &= 40 - 23 \end{aligned}$$

These constants have been calculated for all Q and can be found in each potPrimes Details section as Q_k. The results are: $Q^{31}_k = 0$, $Q^{37}_k = 1$, $Q^{41}_k = 4$, $Q^{43}_k = 5$, $Q^{47}_k = 9$, $Q^{49}_k = 12$, $Q^{53}_k = 17$, and $Q^{59}_k = 28$.

The expansion removal, in addition to dependence on n, also involves a constant associated with each potPrime: it's Q_{base} value mod C. For those wanting to program this there is no need to actually do this calculation, it is constant for each Q: $Q^{31} \mod 30 = 1$, $Q^{37} \mod 30 = 7$, etc.

The equation below gives identical results to equation 20 on the preceding page and so the two equations are equal, T_T is the target TNumber, and $n \leq n_{\max}$ where $n_{\max}$ is obtained by equation 19 on page 36.

$$\begin{aligned} R &= T_T \mod Q_n - (n+1)\left(Q_{\text{base}} \mod C\right) - Q_k \\ \text{when } R &< 0 \text{ then } R = R + Q_n \end{aligned} \tag{22}$$

This equation will always give a sensible answer even if the answer actually has no meaning at that n, and so it must be rigorously contolled by an appropriate n. A negative R means the equation has given the complementary offset value and so must added back with Q_n to give a positive offset.

I recommend using this equation since it is much simpler and gives the same exact mod offset result.

We will use an example of Q^{47}, $n = 2$, and the target TNumber, T_T, is 94090:

$$\begin{aligned} R &= T_T \mod Q^{47}_2 - (n+1)\left(Q^{47}_{\text{base}} \mod 30\right) - Q^{47}_k \\ R &= 94090 \mod 107 - (n+1)(17) - 9 \\ R &= 37 - (3)17 - 9 = -23 \\ R &= R + 107 = 84 \end{aligned}$$

23.04.2021: In an increasing trend to find simpler, and thereby faster, algorithms to find the mod offset of a potPrime at any TNumber I offer the following. It was discovered not by derivation but by inspection of results of manual offset calculations, and also "unrolls" the potPrimes performing similar actions as the preceding alternative equation. All three of these methods provide exactly the same result.

This equation also uses constants derived from a potPrime's properties: Q_F and Q_f. These constants have been calculated for all Q and can be found in each potPrimes Details section.

The F constant is simply $Q \mod 30$ so: $Q_F^{31} = 1$, $Q_F^{37} = 7$, $Q_F^{41} = 11$, $Q_F^{43} = 13$, $Q_F^{47} = 17$, $Q_F^{49} = 19$, $Q_F^{53} = 23$, and $Q_F^{59} = 29$.

The f constant is $Q_{\text{start}} - Q_0$; e.g., for Q^{31} it is $32 - 31 = 1$. The results are: $Q_f^{31} = 1$, $Q_f^{37} = 8$, $Q_f^{41} = 15$, $Q_f^{43} = 18$, $Q_f^{47} = 26$, $Q_f^{49} = 31$, $Q_f^{53} = 40$, and $Q_f^{59} = 57$.

The equation is, where T_T is the target TNumber and $T_T \geq Q_{\text{start}}$:

$$R = (T_T - Q_f - (Q_F n)) \mod Q_n \tag{23}$$

As before we will use an example of Q^{47}, $n = 2$, and the target TNumber, T_T, is 94090:

$$\begin{aligned}
R &= (T_T - Q_f - (Q_F n)) \mod Q_n \\
R &= (94090 - 26 - (17 * 2)) \mod 107 \\
R &= 94030 \mod 107 \\
R &= 84
\end{aligned}$$

5.5.8 Usage of primes 31-59

The idea of these structures I've described were developed with an eye to study formation and longevity of sextuplet primes. I've already identified several areas of interest regarding these structures, see section Analysis of Potential Primes on page 63.

But they are also useful for obtaining lists of sextuplets and other tuplets. For instance, a Basis-0 analysis for only sextuplets (T_{28} to $T_{215656468}$ range) shows that 1,168 Sextuplets survive in that range, while only 786 survive in the Basis-1 range.

In fact, to verify that all these relationships are true, it was necessary to code a program to do so. What follows can be done for any TNumber individually as needed with a calculator, or can be considered an outline for an algorithm to automatically generate lists of tuplets. I'm assuming here that readers of this section may have very differing levels of math prowess and so will be giving explanatory details and examples. I would suggest, however, that each reader at least skim this paper to get the general ideas, terms, and concepts in grip.

The symbol C is, as always, the constant 30, the length of the Prime Template, n is always ≥ 0.

Step 1: Select a TNumber to test, T_T, where $T_T \geq 32$ (this is where the effects of primes 31-59 start).

You can select this randomly but you will have a very low chance of picking a TNumber that has a Tuplet. For this reason I recommend that a copy of a 29basis file is obtained or generated (section 5.3.11 on page 21) and TNumbers selected from there.

If you don't have a 29basis handy then here are some interesting TNumbers to start with: T_{49} contains a right handed quintuplet, T_{535} a sextuplet, T_{630} a quadruplet, T_{647} a sextuplet.

Step 2: Get the $n_{\max}$ for each potPrime for your selected T_T (equation18 on page 35 [complex] or 19 on page 36 [simple]).

$$n_{\max} = \left\lfloor \frac{\sqrt{T_T C + Q_{\text{adj}}} - Q_{\text{base}}}{C} \right\rfloor$$

For calculator calculations, and even computer programs for that matter, I prefer the simpler form even though it uses a extra potPrime constant. By the way, all constants associated with a potPrime are listed under their respective Detail sections.

An $n_{\max}$ is needed for each potPrime: pP(31), pP(37),.... These n's tell us how many members we need to use in that family. Because of the layered nature of potPrimes the $n_{\max}$ of pP(31) will always be the absolute, or greatest, n; the $n_{\max}$'s of the other potPrimes for that T_T can be equal to it or, at most, 1 less.

If the result, **before flooring**, is negative, then that potPrime is not effective for your chosen T_T, and that potPrime is ignored in the following steps. A result of zero is valid and must be used.

Using Q^{47} as an example, remember that the *adj* value is a constant associated with each prime family and is either 1 or 19, for Q^{47} it is 19:

$$n^{47}_{\max} = \left\lfloor \frac{\sqrt{T_T C + Q^{47}_{\text{adj}}} - Q^{47}_{\text{base}}}{C} \right\rfloor$$

$$n^{47}_{\max} = \left\lfloor \frac{\sqrt{T_T\,(30) + 19} - 47}{30} \right\rfloor$$

Step 3, Looping: You will loop over each potPrime from $n = 0 \rightarrow n_{\max}$ (inclusive).

An $n_{\max}$ of zero is valid and must be iterated. If you are programming an algorithm I recommend looping over all potPrimes at each n, rather than looping through all n's for each potPrime in turn. It is simply more efficient because the smaller the potPrime the more likely it will destroy the Tuplet, and so one avoids some unneccesary processing.

Step 3a: Obtain the effective TNumber, T_{eff}, for each potPrime at each n, equation 17 on page 34, you will use this result in the next step:

$$T_{\text{eff}} = Q_{\text{start}} + 2Q_{\text{base}}n + Cn^2$$

Using Q^{47} as an example, with $n = 3$:

$$\begin{aligned}
T_{\text{eff}} &= Q^{47}_{\text{start}} + 2Q^{47}_{\text{base}}n + Cn^2 \\
T_{\text{eff}} &= 73 + 2\,(47)\,n + Cn^2 \\
T_{\text{eff}} &= 73 + 94(3) + 30(3)^2 \\
T_{\text{eff}} &= 625
\end{aligned}$$

Step 3b: For each potPrime at each n get the relative offset to your chosen T_T at that n. There are three equation choices for this step, the first uses TNumber structures to obtain the offset and was derived from first principles (equation 20 on page 37). The second, equation 22 on page 38, uses a regular arithmetic mod expression that is adjusted to give back the offset. And the third, equation 23 on page 39, derived from hand calculations is the simplest and fastest. Use the one you like best, all results are mod based, i.e., starting from 0. You will use this R result in step 3c.

Examples are given and each will use Q^{59}, $n = 2$, and the target TNumber, T_T, is 94090

Step 3b-1: The first equation. The $\mathsf{T}\,(Q, n)$ is our result from step 3a which will then be symbolized by T_{eff}:

$$R = T_T - \left(\left\lfloor \frac{T_T - \mathsf{T}\,(Q, n)}{Q_{\text{base}} + Cn} \right\rfloor (Q_{\text{base}} + Cn) + \mathsf{T}\,(Q, n)\right)$$

Example, $T_T \geq T_{\text{eff}}$ and $n \geq 0$:

$$\begin{aligned}
R &= T_T - \left(\left\lfloor \frac{T_T - T_{\text{eff}}}{Q^{59}_{\text{base}} + Cn} \right\rfloor \left(Q^{59}_{\text{base}} + Cn\right) + T_{\text{eff}}\right) \\
R &= 94090 - \left(\left\lfloor \frac{94090 - 472}{59 + 30n} \right\rfloor (59 + 30n) + 472\right) \\
R &= 94090 - \left(\left\lfloor \frac{93618}{119} \right\rfloor (119) + 472\right) \\
R &= 94090 - (786\,(119) + 472) \\
R &= 94090 - (93534 + 472) \\
R &= 84
\end{aligned}$$

Step 3b-2: The second equation, $n \geq 0$:

$$R = T_T \mod Q_n - (n+1)\left(Q_{\text{base}} \mod C\right) - Q_k$$

Example:

$$\begin{aligned}
R &= T_T \mod Q_n^{59} - (n+1)\left(Q_{\text{base}}^{59} \mod C\right) - Q_k^{59}\\
R &= 94090 \mod Q_2^{59} - (2+1)(59 \mod 30) - 28\\
R &= 94090 \mod 119 - (3)(29) - 28\\
R &= 80 - 87 - 28 = -35 \text{ it's negative so...}\\
R &= -35 + 119\\
R &= 84
\end{aligned}$$

Step 3c: Use the R from the last step to test against your chosen T_T.

This process is detailed in section Using Potential Prime Effect Lookup Tables on page 30. Each potPrime has an effect lookup table with 6 entries. Each lookup is adjusted for inflation using the current n, and the result at that point is compared to your R. Depending on the comparison your T_T is either left untouched, altered, or destroyed.

Here is Q^{47}'s lookup table:

c	q	Q^{47} Effect	Template crossing, mod
0	0	X	c24
3	2	⊢	c28
19	12	X	c18
22	14	X	c22
38	24	⊣	c12
41	26	X	c16

Each row in turn is passed through the lookup equation (14 on page 30) to return a crossing, i, and then i is compared to R:

$$i = c + qn$$

If $R = i$ then take the Effect from that row and apply symbol math to find out how it affects your T_T (section 5.4 on page 23). The comparison logic is explained in detail in the above referenced Using Potential Prime Effect Lookup Tables section, but in summary you test rows until $i \geq R$ and make decisions based on that comparison.

Step 4: Convert a successful result into its Tuplet.

If your chosen T_T passes all the tests through all n for all potPrimes then it contains a Tuplet. The process to display its result in human readable form is explained in detail in section Convert successful result into Tuplet members on page 31.

5.5.9 Elephant in the corner

I've already confessed that potPrimes are not necessarily prime numbers. For instance, $Q_o^{49} = 49$ is not. Nor is $Q_2^{31} = 91$.

This means that tests are being made that simply need not be made. Certainly as n grows the number of composite potPrimes will greatly outnumber prime potPrimes, which means time wasted testing potPrimes that have already, in essence, been tested either by the 29basis file (primes 7-29) or by earlier n's of preceeding potPrimes.

It does not hurt to test composite potPrimes, they simply have no effect on the results.

These prime structures built here are not designed to be efficient, they are designed to be complete. Certainly if a simple accurate test is available to test whether a potPrime is prime or not, it does not hurt to use it (as an aside, in the program I wrote to test these structures, I did try to do simple-ish tests to bypass composite potPrimes, all it did was slow the program down).

The purpose of this study is to bring the primes ≥ 31 into order and to tie them together with equations that will perhaps be of use in elucidating properties of sextuplets that shed light on their survival and destruction, and whether they are infinite. These structures are systematically built such that primes do not need to be considered at all: just n needs to be considered.

Performing needless tests is a consequence of all this, and this only applies if you are going to program these structures into an application.

Yes, there is an elephant in the corner, but she is very friendly.

5.5.10 General notes regarding prime 31-59 Details

The key data for the potPrimes is the Effect Lookup Table. They are used to test the effect at $c + qn$ (see Using Potential Prime Effect Lookup Tables on page 30). Lookup tables are constant for each potPrime and are derived from the natural progression data (Defining properties of potential primes on page 29).

The Q_k values are for use with the (simpler) arithmetic mod offset calculation (page 38). The F value simply reflects the potPrime's value minus the interval size of 30, and is used in the artificial potPrimes discussion (6.4 on page 76).

The Q_F and Q_f are two constants that are used with a recently derived alternative mod result at offset algorithm equation 23 on page 39.

The critical section equation is also a constant for each potPrime, given any n it gives back the length, in TNumbers, from that potPrime's effective start to the *next* potPrime's effective start. See 6.2 on page 65 for the details.

Prime details and natural progressions are also presented for completeness and reference, but have no particular use in discovering Tuplets if that is your main interest. They can be of interest when analyzing the prime structures developed in this study, or for deriving new relationships.

5.5.11 Prime 31 Details

This data applies to 31, 61, 91,...; that is, where Q is any potential prime, $F = (Q mod 30) = 1$.

$Q^{31}_{\text{base}} = Q^{31}_0 \rightarrow$	$\left(Q^{31}_0\right)^2 \rightarrow$	Q^{31}_{start}
31	961	T_{32} (begins at 955)

$Q^{31}_F = \left(Q^{31}_0 mod 30\right)$	$Q^{31}_f = Q^{31}_{\text{start}} - Q^{31}_0$	Q^{31}_{adj}	Q^{31}_k
$31 mod 30 = 1$	$32 - 31 = 1$	$961 \mod 30 = 1$	0

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 12n + 13$

Effect Lookup Table for Q^{31}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{31} Effect	Template crossing, mod
6	6	┫	c12
10	10	X	c16
12	12	X	c18
16	16	X	c22
18	18	X	c24
22	22	┣	c28

Natural progression Q^{31}:

Q 31a	c6	c7	c8	c9	c10	c11	c12	c13	c14	c15	→31b
Effect	●	●	●	●	●	●	┫	●	●	●	→
T_n	T_{32}	T_{33}	T_{34}	T_{35}	T_{36}	T_{37}	T_{38}	T_{39}	T_{40}	T_{41}	→
T_{exp}	955	985	1015	1045	1075	1105	1135	1165	1195	1225	→
c	0	1	2	3	4	5	6	7	8	9	→
q	0	1	2	3	4	5	6	7	8	9	→

31b	c16	c17	c18	c19	c20	c21	c22	c23	c24	c25	→31c
...	X	●	X	●	●	●	X	●	X	●	→
...	T_{42}	T_{43}	T_{44}	T_{45}	T_{46}	T_{47}	T_{48}	T_{49}	T_{50}	T_{51}	→
...	1255	1285	1315	1345	1375	1405	1435	1465	1495	1525	→
c	10	11	12	13	14	15	16	17	18	19	→
q	10	11	12	13	14	15	16	17	18	19	→

31c	c26	c27	c28	c29	c0	c1	c2	c3	c4	c5
...	●	●	⊢	●	o●	●	●	●	●	●
...	T_{52}	T_{53}	T_{54}	T_{55}	T_{56} T_{57}	T_{58}	T_{59}	T_{60}	T_{61}	T_{62}
...	1555	1585	1615	1645	1675 1705	1735	1765	1795	1825	1855
c	20	21	22	23	25	26	27	28	29	30
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{31}, 1 skip space:

Q^{31} CrossNum	Effect	Next Crossing
o c0	●	→c1
c1	●	→c2
c2	●	→c3
c3	●	→c4
c4	●	→c5
c5	●	→c6
c6	●	→c7
c7	●	→c8
c8	●	→c9
c9	●	→c10
c10	●	→c11
c11	●	→c12
c12	⊣	→c13
c13	●	→c14
c14	●	→c15
c15	●	→c16
c16	X	→c17
c17	●	→c18
c18	X	→c19
c19	●	→c20
c20	●	→c21
c21	●	→c22
c22	X	→c23
c23	●	→c24
c24	X	→c25
c25	●	→c26
c26	●	→c27
c27	●	→c28
c28	⊢	→c29
c29	●	→c0

5.5.12 Prime 37 Details

This data applies to 37, 67, 97,...; that is, where Q is any potential prime, $F = (Q mod 30) = 7$.

$Q^{37}_{\text{base}} = Q^{37}_0 \rightarrow$	$(Q^{37}_0)^2 \rightarrow$	Q^{37}_{start}
37	1369	T_{45} (begins at 1345)

$Q^{37}_F = (Q^{37}_0 mod 30)$	$Q^{37}_f = Q^{37}_{\text{start}} - Q^{37}_0$	Q^{37}_{adj}	Q^{37}_k
$37 mod 30 = 7$	$45 - 37 = 8$	$1369 \mod 30 = 19$	1

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 8n + 11$

Effect Lookup Table Q^{37}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{37} Effect	Template crossing, mod
0	0	X	c24
5	4	X	c22
15	12	X	c18
20	16	X	c16
27	22	⊢	c28
30	24	⊣	c12

Natural progression Q^{37}:

Q 37a	c24	c1	c8	c15	c22	c29	c6	c13	c20	c27	→37b
Effect	X	o●	●	●	X	●	o●	●	●	●	→
T_n	T_{45}	T_{46} T_{47}	T_{48}	T_{49}	T_{50}	T_{51}	T_{52} T_{53}	T_{54}	T_{55}	T_{56}	→
$T_{\exp}$	1345	1375 1405	1435	1465	1495	1525	1555 1585	1615	1645	1675	→
c	0	2	3	4	5	6	8	9	10	11	→
q	0	1	2	3	4	5	6	7	8	9	→

37b	c4	c11	c18	c25	c2	c9	c16	c23	c0	c7	→37c
...	o●	●	X	●	o●	●	X	●	o●	●	→
...	T_{57} T_{58}	T_{59}	T_{60}	T_{61}	T_{62} T_{63}	T_{64}	T_{65}	T_{66}	T_{67} T_{68}	T_{69}	→
...	1705 1735	1765	1795	1825	1855 1885	1915	1945	1975	2005 2035	2065	→
c	13	14	15	16	18	19	20	21	23	24	→
q	10	11	12	13	14	15	16	17	18	19	→

37c	c14	c21	c28	c5	c12	c19	c26	c3	c10	c17
...	●	●	⊢	o●	⊣	●	●	o●	●	●
...	T_{70}	T_{71}	T_{72}	T_{73} T_{74}	T_{75}	T_{76}	T_{77}	T_{78} T_{79}	T_{80}	T_{81}
...	2095	2125	2155	2185 2215	2245	2275	2305	2335 2365	2395	2425
c	25	26	27	29	30	31	32	34	35	36
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{37}, 7 skip spaces:

Q^{37} CrossNum	Effect	Next Crossing
o c0	●	→c7
o c1	●	→c8
o c2	●	→c9
o c3	●	→c10
o c4	●	→c11
o c5	●	→c12
o c6	●	→c13
c7	●	→c14
c8	●	→c15
c9	●	→c16
c10	●	→c17
c11	●	→c18
c12	⊣	→c19
c13	●	→c20
c14	●	→c21
c15	●	→c22
c16	X	→c23
c17	●	→c24
c18	X	→c25
c19	●	→c26
c20	●	→c27
c21	●	→c28
c22	X	→c29
c23	●	→c0
c24	X	→c1
c25	●	→c2
c26	●	→c3
c27	●	→c4
c28	⊢	→c5
c29	●	→c6

5.5.13 Prime 41 Details

This data applies to 41, 71, 101,...; that is, where Q is any potential prime, $F = (Q mod 30) = 11$.

$Q^{41}_{\text{base}} = Q^{41}_0 \rightarrow$	$(Q^{41}_0)^2 \rightarrow$	Q^{41}_{start}
41	1681	T_{56} (begins at 1675)

$Q^{41}_F = (Q^{41}_0 mod 30)$	$Q^{41}_f = Q^{41}_{\text{start}} - Q^{41}_0$	Q^{41}_{adj}	Q^{41}_k
$41 mod 30 = 11$	$56 - 41 = 15$	$1681 \mod 30 = 1$	4

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 4n + 5$

Effect Lookup Table Q^{41}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{41} Effect	Template crossing, mod
2	2	⊢	c28
8	6	⊣	c12
16	12	X	c18
24	18	X	c24
27	20	X	c16
35	26	X	c22

Natural progression Q^{41}.

NOTE: this is the first time you see a spacer added at the *end* of the natural progression. When mapping these by hand it appears that the spacer is placed before the first crossing, here c6. But this is illusion because the first crossing is starting from the family's starting Template. Placing this phantom spacer at the end is correct:

Q 41a	c6	c17	c28	c9	c20	c1	c12	c23	c4	c15	→41b
Effect	●	●	⊢	o●	●	o●	⊣	●	o●	●	→
T_n	T_{56}	T_{57}	T_{58}	T_{59} T_{60}	T_{61}	T_{62} T_{63}	T_{64}	T_{65}	T_{66} T_{67}	T_{68}	→
T_{exp}	1675	1705	1735	1765 1795	1825	1855 1885	1915	1945	1975 2005	2035	→
c	0	1	2	4	5	7	8	9	11	12	→
q	0	1	2	3	4	5	6	7	8	9	→

41b	c26	c7	c18	c29	c10	c21	c2	c13	c24	c5	→41c
...	●	o●	X	●	o●	●	o●	●	X	o●	→
...	T_{69}	T_{70} T_{71}	T_{72}	T_{73}	T_{74} T_{75}	T_{76}	T_{77} T_{78}	T_{79}	T_{80}	T_{81} T_{82}	→
...	2065	2095 2125	2155	2185	2215 2245	2275	2305 2335	2365	2395	2425 2455	→
c	13	15	16	17	19	20	22	23	24	26	→
q	10	11	12	13	14	15	16	17	18	19	→

41c	c16	c27	c8	c19	c0	c11	c22	c3	c14	c25
...	X	●	o●	●	o●	●	X	o●	●	●o
...	T_{83}	T_{84}	T_{85} T_{86}	T_{87}	T_{88} T_{89}	T_{90}	T_{91}	T_{92} T_{93}	T_{94}	T_{95} T_{96}
...	2485	2515	2545 2575	2605	2635 2665	2695	2725	2755 2785	2815	2845 2875
c	27	28	30	31	33	34	35	37	38	40
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{41}, 11 skip spaces:

Q 41 CrossNum	Effect	Next Crossing
o c0	●	→c11
o c1	●	→c12
o c2	●	→c13
o c3	●	→c14
o c4	●	→c15
o c5	●	→c16
o c6	●	→c17
o c7	●	→c18
o c8	●	→c19
o c9	●	→c20
o c10	●	→c21
c11	●	→c22
c12	⊣	→c23
c13	●	→c24
c14	●	→c25
c15	●	→c26
c16	X	→c27
c17	●	→c28
c18	X	→c29
c19	●	→c0
c20	●	→c1
c21	●	→c2
c22	X	→c3
c23	●	→c4
c24	X	→c5
c25	●	→c6
c26	●	→c7
c27	●	→c8
c28	⊢	→c9
c29	●	→c10

5.5.14 Prime 43 Details

This data applies to 43, 73, 103,...; that is, where Q is any potential prime, $F = (Q mod 30) = 13$.

$Q^{43}_{\text{base}} = Q^{43}_0 \rightarrow$	$\left(Q^{43}_0\right)^2 \rightarrow$	Q^{43}_{start}
43	1849	T_{61} (begins at 1825)

$Q^{43}_F = (Q^{43}_0 mod 30)$	$Q^{43}_f = Q^{43}_{\text{start}} - Q^{43}_0$	Q^{43}_{adj}	Q^{43}_k
$43 mod 30 = 13$	$61 - 43 = 18$	$1849 \mod 30 = 19$	5

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 8n + 12$

Effect Lookup Table Q^{43}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{43} Effect	Template crossing, mod
0	0	X	c24
6	4	X	c16
9	6	┤	c12
23	16	X	c22
26	18	X	c18
40	28	├	c28

Natural progression Q^{43}.

Q 43a	c24	c7	c20	c3	c16	c29	c12	c25	c8	c21	→43b
Effect	X	o●	●	o●	X	●	o ┤	●	o●	●	→
T_n	T_{61}	T_{62} T_{63}	T_{64}	T_{65} T_{66}	T_{67}	T_{68}	T_{69} T_{70}	T_{71}	T_{72} T_{73}	T_{74}	→
T_{exp}	1825	1855 1885	1915	1945 1975	2005	2035	2065 2095	2125	2155 2185	2215	→
c	0	2	3	5	6	7	9	10	12	13	→
q	0	1	2	3	4	5	6	7	8	9	→

43b	c4	c17	c0	c13	c26	c9	c22	c5	c18	c1	→43c
...	o●	●	o●	●	●	o●	X	o●	X	o●	→
...	T_{75} T_{76}	T_{77}	T_{78} T_{79}	T_{80}	T_{81}	T_{82} T_{83}	T_{84}	T_{85} T_{86}	T_{87}	T_{88} T_{89}	→
...	2245 2275	2305	2335 2365	2395	2425	2455 2485	2515	2545 2575	2605	2635 2665	→
c	15	16	18	19	20	22	23	25	26	28	→
q	10	11	12	13	14	15	16	17	18	19	→

43c	c14	c27	c10	c23	c6	c19	c2	c15	c28	c11
...	●	●	o●	●	o●	●	o●	●	⊢	o●
...	T_{90}	T_{91}	T_{92} T_{93}	T_{94}	T_{95} T_{96}	T_{97}	T_{98} T_{99}	T_{100}	T_{101}	T_{102} T_{103}
...	2695	2725	2755 2785	2815	2845 2875	2905	2935 2965	2995	3025	3055 3085
c	29	30	32	33	35	36	38	39	40	42
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{43}, 13 skip spaces:

Q 43 CrossNum	Effect	Next Crossing
o c0	●	→c13
o c1	●	→c14
o c2	●	→c15
o c3	●	→c16
o c4	●	→c17
o c5	●	→c18
o c6	●	→c19
o c7	●	→c20
o c8	●	→c21
o c9	●	→c22
o c10	●	→c23
o c11	●	→c24
o c12	⊣	→c25
c13	●	→c26
c14	●	→c27
c15	●	→c28
c16	X	→c29
c17	●	→c0
c18	X	→c1
c19	●	→c2
c20	●	→c3
c21	●	→c4
c22	X	→c5
c23	●	→c6
c24	X	→c7
c25	●	→c8
c26	●	→c9
c27	●	→c10
c28	⊢	→c11
c29	●	→c12

5.5.15 Prime 47 Details

This data applies to 47, 77, 107,...; that is, where Q is any potential prime, $F = (Q mod 30) = 17$.

$Q^{47}_{\text{base}} = Q^{47}_0 \rightarrow$	$(Q^{47}_0)^2 \rightarrow$	Q^{47}_{start}
47	2209	T_{73} (begins at 2185)

$Q^{47}_F = (Q^{47}_0 mod 30)$	$Q^{47}_f = Q^{47}_{\text{start}} - Q^{47}_0$	Q^{47}_{adj}	Q^{47}_k
$47 mod 30 = 17$	$73 - 47 = 26$	$2209 \mod 30 = 19$	9

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 4n + 7$

Effect Lookup Table Q^{47}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{47} Effect	Template crossing, mod
0	0	X	c24
3	2	⊢	c28
19	12	X	c18
22	14	X	c22
38	24	⊣	c12
41	26	X	c16

Natural progression Q^{47}.

Q 47a	c24	c11	c28	c15	c2	c19	c6	c23	c10	c27	→47b
Effect	X	o●	⊢	o●	o●	●	o●	●	o●	●	→
T_n	T_{73}	T_{74} T_{75}	T_{76}	T_{77} T_{78}	T_{79} T_{80}	T_{81}	T_{82} T_{83}	T_{84}	T_{85} T_{86}	T_{87}	→
T_{exp}	2185	2215 2245	2275	2305 2335	2365 2395	2425	2455 2485	2515	2545 2575	2605	→
c	0	2	3	5	7	8	10	11	13	14	→
q	0	1	2	3	4	5	6	7	8	9	→

47b	c14	c1	c18	c5	c22	c9	c26	c13	c0	c17	→47c
...	o●	o●	X	o●	X	o●	●	o●	o●	●	→
...	T_{88} T_{89}	T_{90} T_{91}	T_{92}	T_{93} T_{94}	T_{95}	T_{96} T_{97}	T_{98}	T_{99} T_{100}	T_{101} T_{102}	T_{103}	→
...	2635 2665	2695 2725	2755	2785 2815	2845	2875 2905	2935	2965 2995	3025 3055	3085	→
c	16	18	19	21	22	24	25	27	29	30	→
q	10	11	12	13	14	15	16	17	18	19	→

47c	c4	c21	c8	c25	c12	c29	c16	c3	c20	c7
...	o●	●	o●	●	o ┤	●	oX	o●	●	o●
...	T_{104} T_{105}	T_{106}	T_{107} T_{108}	T_{109}	T_{110} T_{111}	T_{112}	T_{113} T_{114}	T_{115} T_{116}	T_{117}	T_{118} T_{119}
...	3115 3145	3175	3205 3235	3265	3295 3325	3355	3385 3415	3445 3475	3505	3535 3565
c	32	33	35	36	38	39	41	43	44	46
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{47}, 17 skip spaces:

Q 47 CrossNum	Effect	Next Crossing
o c0	●	→c17
o c1	●	→c18
o c2	●	→c19
o c3	●	→c20
o c4	●	→c21
o c5	●	→c22
o c6	●	→c23
o c7	●	→c24
o c8	●	→c25
o c9	●	→c26
o c10	●	→c27
o c11	●	→c28
o c12	┤	→c29
o c13	●	→c0
o c14	●	→c1
o c15	●	→c2
o c16	X	→c3
c17	●	→c4
c18	X	→c5
c19	●	→c6
c20	●	→c7
c21	●	→c8
c22	X	→c9
c23	●	→c10
c24	X	→c11
c25	●	→c12
c26	●	→c13
c27	●	→c14
c28	├	→c15
c29	●	→c16

5.5.16 Prime 49 Details

This data applies to 49, 79, 109,...; that is, where Q is any potential prime, $F = (Q mod 30) = 19$.

$Q^{49}_{\text{base}} = Q^{49}_0 \rightarrow$	$\left(Q^{49}_0\right)^2 \rightarrow$	Q^{49}_{start}
49	2401	T_{80} (begins at 2395)

$Q^{49}_F = (Q^{49}_0 mod 30)$	$Q^{49}_f = Q^{49}_{\text{start}} - Q^{49}_0$	Q^{49}_{adj}	Q^{49}_k
$49 mod 30 = 19$	$80 - 49 = 31$	$2401 \mod 30 = 1$	12

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 8n + 13$

Effect Lookup Table Q^{49}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{49} Effect	Template crossing, mod
6	4	X	c22
16	10	X	c16
19	12	X	c24
29	18	X	c18
39	24	⊣	c12
45	28	⊢	c28

Natural progression Q^{49}.

Q 49a	c6	c25	c14	c3	c22	c11	c0	c19	c8	c27	→49b
Effect	●	●	o●	o●	X	o●	o●	●	o●	●	→
T_n	T_{80}	T_{81}	T_{82} T_{83}	T_{84} T_{85}	T_{86}	T_{87} T_{88}	T_{89} T_{90}	T_{91}	T_{92} T_{93}	T_{94}	→
T_{exp}	2395	2425	2455 2485	2515 2545	2575	2605 2635	2665 2695	2725	2755 2785	2815	→
c	0	1	3	5	6	8	10	11	13	14	→
q	0	1	2	3	4	5	6	7	8	9	→

49b	c16	c5	c24	c13	c2	c21	c10	c29	c18	c7	→49c
...	oX	o●	X	o●	o●	●	o●	●	oX	o●	→
...	T_{95} T_{96}	T_{97} T_{98}	T_{99}	T_{100} T_{101}	T_{102} T_{103}	T_{104}	T_{105} T_{106}	T_{107}	T_{108} T_{109}	T_{110} T_{111}	→
...	2845 2875	2905 2935	2965	2995 3025	3055 3085	3115	3145 3175	3205	3235 3265	3295 3325	→
c	16	18	19	21	23	24	26	27	29	31	→
q	10	11	12	13	14	15	16	17	18	19	→

49c	c26	c15	c4	c23	c12	c1	c20	c9	c28	c17
...	●	o●	o●	●	o ⊣	o●	●	o●	⊢	o●o
...	T_{112}	T_{113} T_{114}	T_{115} T_{116}	T_{117}	T_{118} T_{119}	T_{120} T_{121}	T_{122}	T_{123} T_{124}	T_{125}	T_{126} T_{127} T_{128}
...	3355	3385 3415	3445 3475	3505	3535 3565	3595 3625	3655	3685 3715	3745	3775 3805 3835
c	32	34	36	37	39	41	42	44	45	48
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{49}, 19 skip spaces:

Q 49 CrossNum	Effect	Next Crossing
o c0	●	→c19
o c1	●	→c20
o c2	●	→c21
o c3	●	→c22
o c4	●	→c23
o c5	●	→c24
o c6	●	→c25
o c7	●	→c26
o c8	●	→c27
o c9	●	→c28
o c10	●	→c29
o c11	●	→c0
o c12	⊣	→c1
o c13	●	→c2
o c14	●	→c3
o c15	●	→c4
o c16	X	→c5
o c17	●	→c6
o c18	X	→c7
c19	●	→c8
c20	●	→c9
c21	●	→c10
c22	X	→c11
c23	●	→c12
c24	X	→c13
c25	●	→c14
c26	●	→c15
c27	●	→c16
c28	⊢	→c17
c29	●	→c18

5.5.17 Prime 53 Details

This data applies to 53, 83, 113,...; that is, where Q is any potential prime, $F = (Q mod 30) = 23$.

$Q^{53}_{\text{base}} = Q^{53}_0 \rightarrow$	$\left(Q^{53}_0\right)^2 \rightarrow$	Q^{53}_{start}
53	2809	T_{93} (begins at 2785)

$Q^{53}_F = (Q^{53}_0 mod 30)$	$Q^{53}_f = Q^{53}_{\text{start}} - Q^{53}_0$	Q^{53}_{adj}	Q^{53}_k
$53 mod 30 = 23$	$93 - 53 = 40$	$2809 \mod 30 = 19$	17

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 12n + 23$

Effect Lookup Table Q^{53}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{53} Effect	Template crossing, mod
0	0	X	c24
11	6	⊣	c12
14	8	⊢	c28
25	14	X	c16
32	18	X	c18
46	26	X	c22

Natural progression Q^{53}.

Q 53a	c24	c17	c10	c3	c26	c19	c12	c5	c28	c21	→53b
Effect	X	o●	o●	o●	●	o●	o ⊣	o●	⊢	o●	→
T_n	T_{93}	T_{94} T_{95}	T_{96} T_{97}	T_{98} T_{99}	T_{100}	T_{101} T_{102}	T_{103} T_{104}	T_{105} T_{106}	T_{107}	T_{108} T_{109}	→
T_{exp}	2785	2815 2845	2875 2905	2935 2965	2995	3025 3055	3085 3115	3145 3175	3205	3235 3265	→
c	0	2	4	6	7	9	11	13	14	16	→
q	0	1	2	3	4	5	6	7	8	9	→

53b	c14	c7	c0	c23	c16	c9	c2	c25	c18	c11	→53c
...	o●	o●	o●	●	oX	o●	o●	●	oX	o●	→
...	T_{110} T_{111}	T_{112} T_{113}	T_{114} T_{115}	T_{116}	T_{117} T_{118}	$T_{119}T_{120}$	T_{121} T_{122}	T_{123}	T_{124} T_{125}	T_{126} T_{127}	→
...	3295 3325	3355 3385	3415 3445	3475	3505 3535	3565 3595	3625 3655	3685	3715 3745	3775 3805	→
c	18	20	22	23	25	27	29	30	32	34	→
q	10	11	12	13	14	15	16	17	18	19	→

53c	c4	c27	c20	c13	c6	c29	c22	c15	c8	c1
...	o●	●	o●	o●	o ●	●	oX	o●	o●	o●
...	T_{128} T_{129}	T_{130}	T_{131} T_{132}	T_{133} T_{134}	T_{135} T_{136}	T_{137}	T_{138} T_{139}	T_{140} T_{141}	T_{142} T_{143}	T_{144} T_{145}
...	3835 3865	3895	3925 3955	3985 4015	4045 4075	4105	4135 4165	4195 4225	4255 4285	4315 4345
c	36	37	39	41	43	44	46	48	50	52
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{53}, 23 skip spaces:

Q 53 CrossNum	Effect	Next Crossing
o c0	●	→c23
o c1	●	→c24
o c2	●	→c25
o c3	●	→c26
o c4	●	→c27
o c5	●	→c28
o c6	●	→c29
o c7	●	→c0
o c8	●	→c1
o c9	●	→c2
o c10	●	→c3
o c11	●	→c4
o c12	⊣	→c5
o c13	●	→c6
o c14	●	→c7
o c15	●	→c8
o c16	X	→c9
o c17	●	→c10
o c18	X	→c11
o c19	●	→c12
o c20	●	→c13
o c21	●	→c14
o c22	X	→c15
c23	●	→c16
c24	X	→c17
c25	●	→c18
c26	●	→c19
c27	●	→c20
c28	⊢	→c21
c29	●	→c22

5.5.18 Prime 59 Details

This data applies to 59, 89, 119,...; that is, where Q is any potential prime, $F = (Q mod 30) = 29$.

$Q^{59}_{\text{base}} = Q^{59}_0 \rightarrow$	$(Q^{59}_0)^2 \rightarrow$	Q^{59}_{start}
59	3481	T_{116} (begins at 3475)

$Q^{59}_F = (Q^{59}_0 mod 30)$	$Q^{59}_f = Q^{59}_{\text{start}} - Q^{59}_0$	Q^{59}_{adj}	Q^{59}_k
$59 mod 30 = 29$	$116 - 59 = 57$	3481 mod 30 = 1	28

Critical section equation, $n \geq 0$, L is the length of the critical section:
$L = 4n + 8$

Effect Lookup Table Q^{59}, c is crossing effect with respect to Q_{base}, and q is the inflation multiplier with respect to Template length (seeUsing Potential Prime Effect Lookup Tables on page 30).

c	q	Q^{59} Effect	Template crossing, mod
15	8	⊢	c28
23	12	X	c24
27	14	X	c22
35	18	X	c18
39	20	X	c16
47	24	⊣	c12

Natural progression Q^{59}.

Q 59a	c6	c5	c4	c3	c2	c1	c0	c29	c28	c27	→59b
Effect	●	o●	o●	o●	o●	o●	o ●	●	o⊢	o●	→
T_n	T_{116}	T_{117} T_{118}	T_{119} T_{120}	T_{121} T_{122}	T_{123} T_{124}	T_{125} T_{126}	T_{127} T_{128}	T_{129}	T_{130} T_{131}	T_{132} T_{133}	→
$T_{\exp}$	3475	3505 3535	3565 3595	3625 3655	3685 3715	3745 3775	3805 3835	3865	3895 3925	3955 3985	→
c	0	2	4	6	8	10	12	13	15	17	→
q	0	1	2	3	4	5	6	7	8	9	→

59b	c26	c25	c24	c23	c22	c21	c20	c19	c18	c17	→59c
...	o●	o●	oX	o●	oX	o●	o●	o●	oX	o●	→
...	T_{134} T_{135}	T_{136} T_{137}	T_{138} T_{139}	T_{140} T_{141}	T_{142} T_{143}	T_{144} T_{145}	T_{146} T_{147}	T_{148} T_{149}	T_{150} T_{151}	T_{152} T_{153}	→
...	4015 4045	4075 4105	4135 4165	4195 4225	4255 4285	4315 4345	4375 4405	4435 4465	4495 4525	4555 4585	→
c	19	21	23	25	27	29	31	33	35	37	→
q	10	11	12	13	14	15	16	17	18	19	→

59c	c16	c15	c14	c13	c12	c11	c10	c9	c8	c7
...	oX	o●	o●	o●	o ┤	o●	o●	o●	o●	o●o
...	T_{154} T_{155}	T_{156} T_{157}	T_{158} T_{159}	T_{160} T_{161}	T_{162} T_{163}	T_{164} T_{165}	T_{166} T_{167}	T_{168} T_{169}	T_{170} T_{171}	T_{172} T_{173} T_{174}
...	4615 4645	4675 4705	4735 4765	4795 4825	4855 4885	4915 4945	4975 5005	5035 5065	5095 5125	5155 5185 5215
c	39	41	43	45	47	49	51	53	55	58
q	20	21	22	23	24	25	26	27	28	29

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Raw data Q^{59}, 29 skip spaces:

Q 59 CrossNum	Effect	Next Crossing
o c0	●	→c29
o c1	●	→c0
o c2	●	→c1
o c3	●	→c2
o c4	●	→c3
o c5	●	→c4
o c6	●	→c5
o c7	●	→c6
o c8	●	→c7
o c9	●	→c8
o c10	●	→c9
o c11	●	→c10
o c12	┤	→c11
o c13	●	→c12
o c14	●	→c13
o c15	●	→c14
o c16	X	→c15
o c17	●	→c16
o c18	X	→c17
o c19	●	→c18
o c20	●	→c19
o c21	●	→c20
o c22	X	→c21
o c23	●	→c22
o c24	X	→c23
o c25	●	→c24
o c26	●	→c25
o c27	●	→c26
o c28	├	→c27
c29	●	→c28

6 Analysis of Potential Primes

This section is under development. The original publishing in Feb. 2021 was done so that the basic structure of the potPrimes was available for study and to complement the computer code.

There are several more pages of notes that need to be transcribed. They will be published here as time permits.

Check back at the download site for updated pdf.

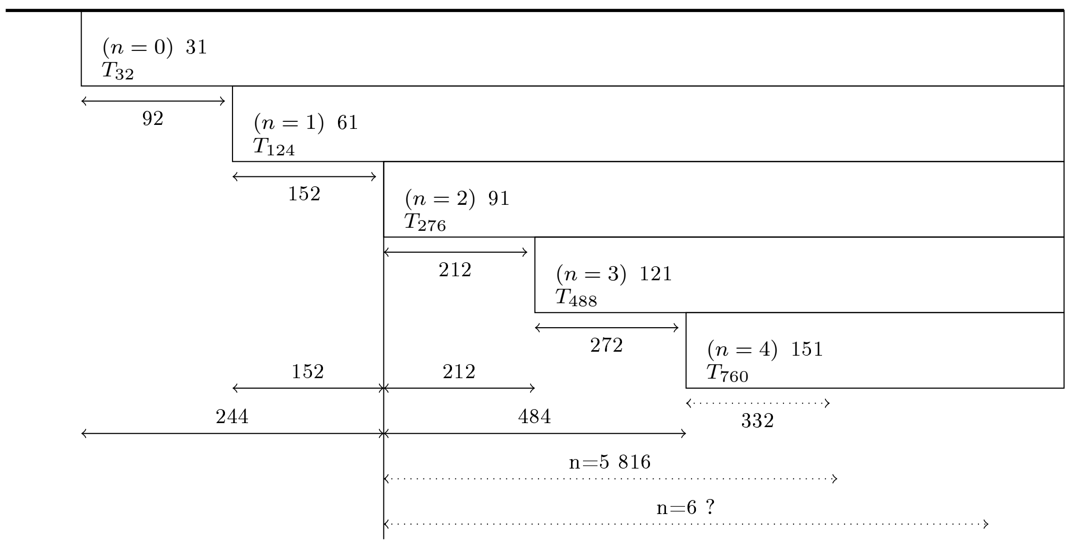
6.1 Critical Sections, within Family

A critical section is the region between squares of suceeding potPrimes; i.e., between their effective TNumbers. There are critical sections between the members of a family (31. . . 61) and between differing families (31. . . 37, 49. . . 53, etc., on page 65).

These regions are important because they are the havens of stability as one moves through the primes. Everything is finalized in these regions and one is assured that conclusions, as regards for example Tuplet survival, in these regions are true. Due to the nature of the layering of potPrimes within Family critical sections are interesting but of limited usefulness for complete analysis, for complete analysis see the between families section.

6.1.1 Critical Sections, within family

The following diagram shows the critical sections of a single family, Q^{31}, from $n = 0 \rightarrow 4+$. The heavy black line at top represents the number line in TNumbers. The diagram is representative and not to scale, in particular not showing the exponential nature of the suceeding effective TNumbers at each n.



At each n is shown the length, in TNumbers, between adjacent potPrime effective TNumbers, that is, their critical lengths. For example., between pP(31) and pP(61) are exactly 92 TNumbers, between pP(61) and pP(91) exactly 152, and so on. These numbers are easily obtained by subtracting the effective TNumber at n from the effective TNumber at $n+1$. The effective TNumber equation 17 on page 34 is shown below:

$$T_{\text{eff}} = Q_{\text{start}} + 2Q_{\text{base}}n + Cn^2$$

However, as the arrows at the bottom of the diagram indicate, it could be useful to be able to determine distances in TNumbers from any effective TNumber we like to any other effective TNumber we like. Or to be able to do math with various lengths such as these. An equation to do so is easily derived from the equation above.

We will first change it to the form below for an easier derivation, T_D is the distance between effective TNumbers that we are searching for, Q_{start} will be noted by T_S, and Q_{base} by p:

$$T_D = T_S + 2pn + Cn^2$$

We now subtract the nth effective TNumber from the $(n+d)$th effective TNumber, which will give us the distance in TNumbers:

$$\begin{aligned}
T_D &= T_S + 2p\,(n+d) + C\,(n+d)^2 - \left(T_S + 2pn + Cn^2\right) \\
T_D &= T_S + 2pn + 2pd + C\left(n^2 + 2nd + d^2\right) - T_S - 2pn - Cn^2 \\
T_D &= T_S + 2pn + 2pd + Cn^2 + 2Cnd + Cd^2 - T_S - 2pn - Cn^2 \\
T_D &= 2pd + Cd^2 + 2Cnd \\
T_D &= d\,(2p + Cd + 2Cn)
\end{aligned}$$

We can see now that equation 2, $x = 2p+C+2Cn$, from the section Interval-izing regions between squares, is actually a special case of the above equation where $d = 1$.

The operative variable in these equations is d, and n takes on a given, fixed role. We will use $\mathbf{N}$ to denote that a chosen n is a fixed point, as $n = 2$ is in the diagram above (the central vertical line), and n is a level that we wish to determine the distance to. This makes d the difference between the n and $\mathbf{N}$ and so $d = n - \mathbf{N}$ where $n \geq 0$ and $\mathbf{N} \geq 0$.

Our derived equation, using the regular symbols:

$$T_D = d\,(2Q_{\text{base}} + Cd + 2C\mathbf{N}) \tag{24}$$

As an example, we will now recreate the distances, which we could call member sections, in the diagram on page 63 using Q^{31}, so $Q_{\text{base}} = 31$. The arrows at the bottom of the diagram are focused on pP(91) where $n = 2$, so $\mathbf{N} = 2$. It is in accordance with the usage case whether the result should be returned as the absolute value or not:

$\mathbf{N}$	n	$d = n - \mathbf{N}$	$d\,(62 + 30d + 60\mathbf{N})$	T_D
2	5	3	$3\,(62 + 90 + 120)$	816
2	4	2	$2\,(62 + 60 + 120)$	484
2	3	1	$1\,(62 + 30 + 120)$	212
2	2	0	$0\,(62 + 0 + 120)$	0
2	1	-1	$-1\,(62 - 30 + 120)$	-152
2	0	-2	$-2\,(62 - 60 + 120)$	-244
2	-1	illegal: $n < 0$	-	-

A within family critical section can now be defined as the length resulting from using equation 24, setting $\mathbf{N}$ to any n we are interested in, and where $d = 1$.

For those who may be interested in finding the offsets of preceding effective TNumbers to any effective TNumber boundare at the n of choice see the appendix section 7.3 on page 78 for a complete discussion.

6.1.2 Ratio of growth of potPrime to the growth of the critical section

How does the growth of a potPrime, Q_n, relate to the growth of a critical section (using $d = 1$ in equation from the last section)?

The relationship between the two are dependent on n:

$$\frac{2p + C + 2Cn}{p + Cn}$$

The variables p and C are constant, so as $n \to \infty$:

$$\frac{k + 2Cn}{k + Cn} \to \frac{2Cn}{Cn} = 2$$

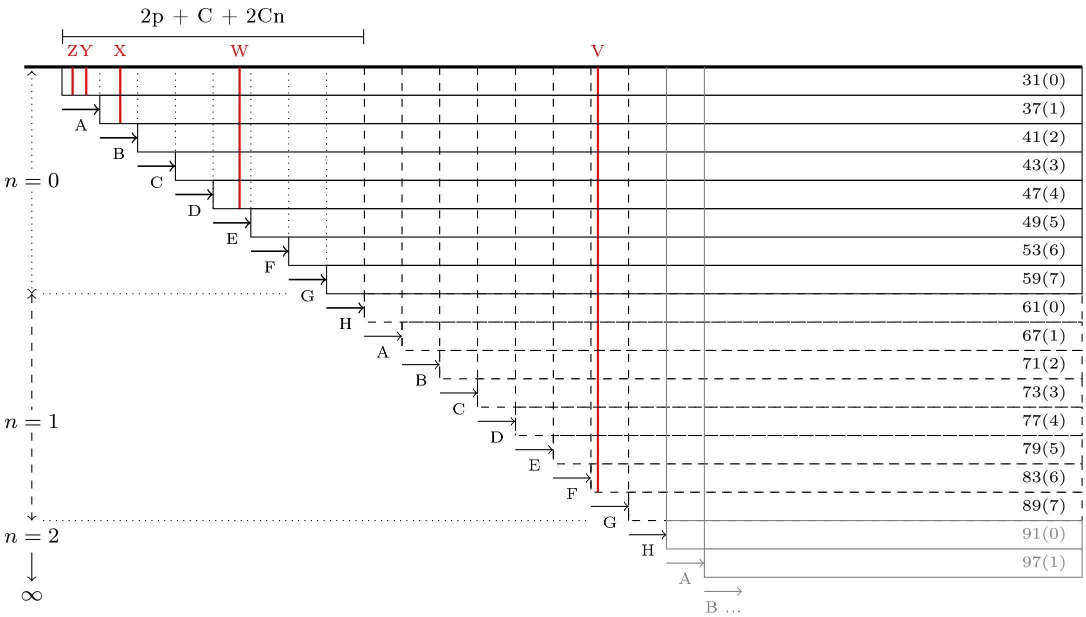
This indicates that there are at least two full cycles of Q_n in the critical section at n.

6.2 True critical sections & critical rectangles, between families

As mentioned there are critical sections between members of a family, and also between differing potPrime families. The latter describe the actual situation faced when analyzing the distribution of sextuplets, and, for this reason, are the “true” critical sections. From here on when refering to “critical sections” it will be regarding between family sections as described in this section.

The diagram below illustrates the idea of the connection between n and each potPrime family by diagrammatically showing the critical sections. The black arrows illustrate the distance, in TNumbers, we want to find. The letters associated with each will be used later to tie each relationship to an equation. The heavy black line at top represents the number line in TNumbers. The diagram is representative and not to scale, as usual not showing the exponential nature of potPrime effective starting positions, and in particular not showing that each critical section has a different length, and that those lengths increase as n increases.

The "A" arrow represents the distance from the effective start of Q_n^{31} to the effective start of Q_n^{37}, "B" similarly the distance from Q_n^{37} to Q_n^{41}, and so on. The numbers in parentheses after each potPrime on the right will be discussed shortly.



The equation at the top of the diagram is there as a reference and brackets the full length of Q_0^{31} to Q_1^{31}; that is, it is the general equation for critical section within a family as discussed in detail in the section prior to this section.

The thick red lines represent TNumbers of interest and will be used later to demonstrate a mapping scheme for critical sections.

The diagram shows how at each n each potPrime family must be evaluated with that n, as n increases the equations are exactly the same and so are continued to be labeled A-H. The other potPrime families divide the within family pP(31) critical section into eight pieces. We will now derive the equations for these true critical sections and will see that their sum will equal $2p + C + 2Cn$ as evaluated for pP(31).

Also note that each critical section defines the horizontal sides of a rectangle whose height is the number of rows at that point, these rectangles will be discussed in detail further on. It will be interesting to note that each rectangle has a very specific shape, at least at large n.

6.2.1 Derivation of critical section equations

As mentioned before critical sections are defined as the distance between effective starts of succeeding Q_n's. We need to build expressions that describe for each potPrime family how far, in TNumbers, is it to the succeeding potPrime family. We will can use the effective start equation: $T_{\text{eff}} = Q_{\text{start}} + 2Q_{\text{base}}n + Cn^2$ (17 on page 34). But first we will simplify its form so it is easier to follow and we will use an associated potPrime constant that simplifies the derived equations.

With T being the starting TNumber (constant for each potPrime), and p the Q_0 value the equation becomes: $T + 2pn + Cn^2$. Here p is simply the base value of the potPrime: 31, 43, 59, etc. But each potPrime base value is nothing more than the sum of 30 plus $p \mod 30$. So $31 = 30 + 1$, $43 = 30 + 13$, $59 = 30 + 29$,...

These mod 30 remainders of the potPrimes are defined as their F values: $Q_F^{31} = 1$, $Q_F^{43} = 13$, $Q_F^{59} = 29$,...

So we will replace p in the equation with $C + F$, here S is the effective start of that potPrime at n, and T is starting TNumber for that potPrime:

$$S = T + 2\left(C + F\right)n + Cn^2$$
$$S = T + 2Cn + 2Fn + Cn^2$$

To find our equations then we only need to select any potPrime we are interested in and subtract its effective start from the next potPrimes effective start, here subscript 2 is the next potPrime's data, ie. its effective start is greater than our selected potPrime effective start, D is the critical section length we are looking for:

$$D = T_2 + 2Cn + 2F_2n + Cn^2 - \left(T_1 + 2Cn + 2F_1n + Cn^2\right)$$
$$D = T_2 + 2Cn + 2F_2n + Cn^2 - T_1 - 2Cn - 2F_1n - Cn^2$$
$$D = 2F_2n - 2F_1n + T_2 - T_1$$

Dropping in the values of these constants from Q^{31} and Q^{37} from any n is fine so we will use $n = 0$. In that case $T_1 = Q_{\text{start}}^{31} = 32$, $F_1 = 1$, $T_2 = Q_{\text{start}}^{37} = 45$, and $F_2 = 7$:

$$D = 2F_2n - 2F_1n + T_2 - T_1$$
$$D = 2\left(7\right)n - 2\left(1\right)n + 45 - 32$$
$$D = 14n - 2n + 13$$
$$D = 12n + 13$$

So $12n + 13$ is a constant for pP(31) and will give the critical distance from its current n to the effective start of pP(37) at the same n. The derivation of the equations for the other potPrimes is the same, except some attention must be paid when deriving it for Q^{59}, if you use $T_2 = 32$ the equations will have negative terms but the result is correct if you derive it with n and $n + 1$ respectively for the 2 subscripts, or you can use the effective start of Q^{31} at $n = 1$ which is equal to 124 and results in its $F = F + C$ and keep the derivation with postive terms. All the constants we've discussed so far can be found listed in each potPrime's details sections.

We can now relate the equations A-H from the diagram with the appropriate equations. This is summarized in the table below:

Diagram	from/to	constants	critical section length	used for Q
A	as $Q^{31} \to Q^{37}$	$2(7)n - 2(1)n + 45 - 32$	$12n + 13$	Q_n^{31} (0)
B	as $Q^{37} \to Q^{41}$	$2(11)n - 2(7)n + 56 - 45$	$8n + 11$	Q_n^{37} (1)
C	as $Q^{41} \to Q^{43}$	$2(13)n - 2(11)n + 61 - 56$	$4n + 5$	Q_n^{41} (2)
D	as $Q^{43} \to Q^{47}$	$2(17)n - 2(13)n + 73 - 61$	$8n + 12$	Q_n^{43} (3)
E	as $Q^{47} \to Q^{49}$	$2(19)n - 2(17)n + 80 - 73$	$4n + 7$	Q_n^{47} (4)
F	as $Q^{49} \to Q^{53}$	$2(23)n - 2(19)n + 93 - 80$	$8n + 13$	Q_n^{49} (5)
G	as $Q^{53} \to Q^{59}$	$2(29)n - 2(23)n + 116 - 93$	$12n + 23$	Q_n^{53} (6)
H	as $Q^{59} \to Q^{31}$	$2(31)n - 2(29)n + 124 - 116$	$4n + 8$	Q_n^{59} (7)
SUM	–	–	$60n + 92$	

We can compare the final sum to the within family critical distance for Q^{31} (shown at the top of the diagram) by substituting in the appropriate values:

$$
\begin{aligned}
2p + C + 2Cn \\
2(31) + 30 + 2(30)n \\
62 + 30 + 60n \\
60n + 92
\end{aligned}
$$

So another way to derive the Q^{59} equation is to subtract the sum of the equations for Q^{31}-Q^{53} which is $56n + 84$ from the result above to give $4n + 8$.

With this we see that all TNumbers in the range are accounted for and we now have powerful tools allowing us to analyze any critical distance we want.

6.2.2 Identifying and mapping critical sections

Examine the diagram on page 66 again and focus on the thick red lines. These represent arbitrary TNumbers and shows that any TNumber lives in a particular critical section and, depending on its position in the number line, requires a particular number of potPrime iterations before we can be certain the TNumber is, or is not, a sextuplet.

Knowing these bounds is central to any analysis that is desired and so the importance of critical sections becomes apparent: they are the final deciders of a Prime Template's fate.

Each n encompasses all eight potPrimes. A convention used here, and useful for computer programmers, is that the digits 0-7 represent the potPrime families Q^{31}-Q^{59} respectively. Hence the numbers in parentheses following the potPrime indicators in the diagram correspond to this scheme.

The Z and Y TNumbers in the diagram both live in $n = 0$ at the Q^{31} level. From our preceding analysis we know that the critical distance for Q^{31} is $12n + 13$ which at $n = 0$ then equals 13. In this critical section live 13 TNumbers, Z and Y are but two of them. However all thirteen share this "address".

The convention I use for this is an "n:m" format (n, m are integers), which I call the "Critical Section ID", or csID for short, where n is our usual n, and m is the potPrime indicator 0-7 as discussed above. And so the address for Z and Y, and the other 11 TNumbers in this critical section is 0:0. For X the address is 0:1, W is 0:4, and V is 1:6.

The nice thing about this convention is that n is always visible and not an imponderable abstraction. Our n indicates not only the depth we need to go but also directly the expansion of $Q's$ at that csID.

In actuality of course, the critical sections are rows each of which is a uniqe number, and this is also useful allowing us to calculate rectangles for each critical section. The addresses above convert thusly:

csID	0:0	0:1	0:4	1:6
Unique number	1	2	5	15

Converting between unique numbers and csID's is as follows:

First getting a unique number from a csID, N is a critical section row/unique number, and using n:m format where $m = 0 \ldots 7$:

$$N = 8n + m + 1 \tag{25}$$

Calculating a csID from a given unique number, same properties as above:

$$n = (N - 1) \text{ div } 8$$
$$m = (N - 1) \mod 8$$

As regards csID math:

- There is no 0 or negative csID. The csID 0:0 is equivalent to 1. Therefore when subtracting csID's the subtracted csID (subtrahend) must be less than the csID being subtracted from (minuend); eg. 0:7 minus 0:6 is valid and would equal 0:0, 0:7 minus 0:7, 0:7 minus 1:0, etc are invalid and undefined.

- When incrementing csID's keep in mind that m is modular 8 and so for each increase of $m = 7 \rightarrow m = 0$ requires that n is incremented; eg., 0:7 (8) + 0:0 (1) = 1:0 (9).

Other common tasks for csID's would be to determine the csID given a TNumber, and to get the starting and ending TNumbers (inclusive) from a given csID:

To get a csID from a TNumber one uses the "get n" equation (19 on page 36) and get the n for each potPrime at the target TNumber. The n in n:m will be the maximum n found, and the m will be the largest corresponding 0-7 value for the potPrime family that had the maximum n.

To get the starting and ending (inclusive) TNumbers for a given csID use the getting effective TNumber equation (17 on page 34). Use the n from the given csID's n:m as the n to that equation and use the Q values from the corresponding m (0-7) potPrime family, this gives the "from" TNumber. Next increment the given n:m by 0:0 (equivalent to 1), and perform the same operations using the newly, properly, incremented n:m. Subtract 1 from that result to give the inclusive "to" TNumber.

6.2.3 A practical csID example

The first sextuplet (after 97, 101, 103...) is found in TNumber 535 (T_{535}), its sextuplet values are: 16057, 16061, 16063, 16067, 16069, 16073.

To find the csID from T_{535} we get the max n from each potPrime at TNumber 535:

Q	m	max n	potPrimes evaluated by $n = 0 \rightarrow$ max n
Q^{31}	0	3	31, 61, 91,121
Q^{37}	1	2	37, 67, 97
Q^{41}	2	2	41, 71, 107
Q^{43}	3	2	43, 73, 103
Q^{47}	4	2	47, 77, 107
Q^{49}	5	2	49, 79, 109
Q^{53}	6	2	53, 83, 113
Q^{59}	7	2	59, 89, 119

The csID in n:m format uses the greatest n from above and the m from the last Q to have that max n, so the csID for T_{535} is 3:0. Its unique row number is 25. To determine that T_{535} was a clear channel from top to bottom (ie. it is a sextuplet) all potPrimes were evaluated from $n = 0 \ldots 2$ and finally a last evaluation at Q_3^{31} at $n = 3$. It shows that potPrimes from 31. . . 121 needed to be checked to ensure we had exhausted all potentially destructive potPrimes.

To get the critical section length: since $m = 0$ it indicates that we will use critical section equation of Q^{31}: $12n + 13 = 12\,(3) + 13 = 36 + 13 = 49$.

The TNumber at the beginning of this critical section is the effective TNumber for Q^{31} at $n = 3$ which is 488, this is our "from" TNumber.

Now we increment csID 3:0 which gives us 3:1, the 1 indicates that our next potPrime is Q^{37}. The effective TNumber for Q^{37} at $n = 3$ is 537, which is the non-inclusive "to" TNumber.

Subtract 1 from 537, to make the "to" inclusive, and the range of the critical section 3:0 are the TNumbers 488-536 inclusive.

6.2.4 Critical rectanges and their shapes

The critical sections depicted in the diagram on page 66 are woefully inaccurate. For the purposes of explanation they were drawn with equal lengths but this is far from the case. First each critical section is governed by a potPrime specific critical section equation which increases without limit as n grows. The actual beginning TNumbers for each potPrime critical section are based on the square of the potPrime and so exponentially increase the distance between each other. Critical section distribution in actuality is a never ending avalanche of sliding and growing sections.

And each critical section defines the bounds of a rectangle, the "critical rectangle", bound to each csID. The horizontal sides are defined by the critical section length, and the vertical sides, by its height, which is the csID's unique row number in the infinity of potPrimes.

The relations of the sides of the rectangle of the critical sections are based on the difference between the subsequent Q_0's which divides them into 3 size classes:

	31→	37→	41→	43→	47→	49→	53→	59→	61
distance to next	6	4	2	4	2	4	6	2	-

This gives us the following 3 size tables:

size 6	critical section equation
Q^{31}	$12n + 13$
Q^{53}	$12n + 23$
generic	$12n + k$

size 4	critical section equation
Q^{37}	$8n + 11$
Q^{43}	$8n + 12$
Q^{49}	$8n + 13$
generic	$8n + k$

size 2	critical section equation
Q^{41}	$4n + 5$
Q^{47}	$4n + 7$
Q^{59}	$4n + 8$
generic	$4n + k$

Each complete n holds all eight potPrimes and so the height of a critical rectangle is $8n + \{0 \ldots 7\} + 1$ or $8n + k$.

As n grows the small k constants become insignificant and we see the shapes of the rectangles by dividing the critical length by the height at that critical section:

size 6	$\frac{12n}{8n} = 1.5$
size 4	$\frac{8n}{8n} = 1$
size 2	$\frac{4n}{8n} = 0.5$

Using analysis routines in my prime sextuplet package I looked for critical lengths as close to $215,656,441$ as possible. This the is length of the 29basis and it is here where a single critical section will encompass an entire Basis-#. The results for Q^{31}, where the csID will be 17971369:0, gives the true critical length at that CS as 215656441, the height as 143770952, and the ratio of the two as 1.5000000904216033845 as expected. Testing all the other potPrimes produced the expected results as outlined in the table above.

Note for those investigating this relationship: The size classes will arrive at the targeted critical length at different times due to the nature of their different growth rates. In general pP(31), pP(53) will be the first to achieve a targeted critical length and pP(41), pP(47), pP(59) the last to do so. For example, at the same critical length, the n for Q^{53} is $17,971,368$ while the n for Q^{59} is $53,914,108$. By the time Q^{59} has finally achieved the desired critical length, Q^{53} will have grown to be larger.

A bird's eye view of the potPrime's critical rectangles at infinity will have a repeating pattern that looks approximately like this:

Q^{31}	Q^{37}	Q^{41}	Q^{43}	Q^{47}	Q^{49}	Q^{53}	Q^{59}

Of course, each critical section side height is 1 row deeper than the previous one, so as one goes even "farther" out each of the critical section heights will also grow to infinity. I wonder if there is a sextuplet out there??

6.2.5 A word about the non-prime potPrimes

As n gets larger and larger as we move further out into the jungle of infinite primes, more and more of the potPrimes are non-prime. For example, the size of potPrime Q_2^{31} is 91 which is not a prime number.

There will be a critical section at 2:0 based on this potPrime. But since 91 is not prime having this critical section, or evaluating it, or any other action will have no effect on whether a TNumber will survive to sextuplet-hood or not. This is because sextuplet-hood for 91 has already been determined by 7 and 13, the factors of 91. Any non-prime potPrime will be factored by either the 29basis or a smaller preceding potPrime.

Since the critical section at 2:0 is "useless" we can simply ignore it and add its length to the previous prime potPrime increasing its true critical section by that length. This will result in a "true" true critical section. This can be chained along for, at huge n, millions of non-prime potPrimes until finally another prime potPrime takes over. This would change the shapes of critical sections completely, of course.

So aren't we wasting time considering non-prime potPrimes? For several reasons I do not believe so.

If there are regularities to be found in the structure of the potPrimes then the non-prime potPrimes may be crucial to discovering and understanding them.

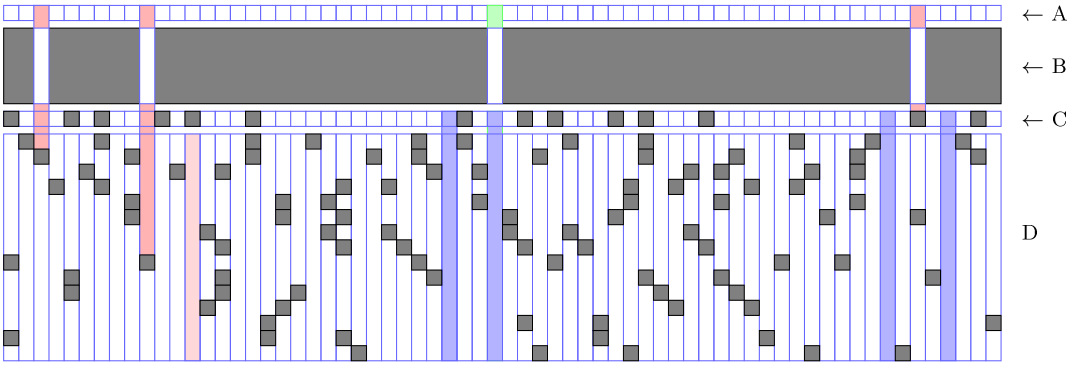
Including them is natural, no special rules or algorithms are needed, they form naturally from the potPrimes equations and structures.

But probably the best reason for not worrying about non-primes is for reasons of practicality. Were we to filter non-primes then it would require an absolutely accurate and wonderfully quick way to determine if a Q_n is prime or not. Unfortunately, neither of these exist. Out at very large n it would be basically impossible to make the decision whether to process that potPrime or to collapse it.

So it is good to keep in mind that we could "collapse" non-prime potPrimes into a "true" true critical lengths if we want to and have the computer time to do so. Maybe it could tell us something new about the underlying rules of sextuplets, but, in general, it is completely impractical.

6.3 A visualization of the sextuplet goal

We start by focusing on a portion of a single critical section to see exactly how a pristine prime sextuplet template can survive. The diagram below is only approximately accurate (with the exception of block C), and not to scale but is intended to demonstrate the typical situation within a critical section.



Block A is a representation of the 2,3,5 primes; ie., TNumbers. Each square is a Prime Template (TNumber) having a pristine copy of the sextuplet ready to run the guantlet. Each of these is tested downward throught the block layers. Most cannot be sextuplets because they are stopped by the gray portion of B.

Block B represents the 29basis. The white paths are the "pre-approved for sextuplet-hood" channels created when we applied the primes 7-29 to the T-Numberline and created the 215+ million length 29basis. These "29pipe" channels are the only possible locations where a sextuplet can exist. The actual numerical, TNumber, positions of the 29pipes are important only if one is interested in generating lists of sextuplets which, if so, is done using the 29basis files and the methods discussed throughout this document. Otherwise, we only need to know this: 29pipes exist, they occur in a pattern that repeats to infinity, and each cycle contains a a bit under 2 million 29pipes.

Block C represents Q_0^{31}. Each gray square is a position in the potPrime's natural progression where it effects, that is changes, a pristine sextuplet coming from above, these "destroy" the incoming sextuplet's form. The clear squares are "passes" which means an incoming sextuplet progresses through that position unchanged. The representation of the strikes and passes is accurate and shows a bit over two complete cycles of length 31.

Block D is the rest of the potPrimes from Q_0^{37} on and the diagram shows, approximately accurately, the patterns of potPrimes Q_0^{37} to Q_1^{59}. Each row in D simulates the strikes and passes of the respective potPrime row through this critical section.

We have one sextuplet in the diagram, the green square in A. It occurs when A, B, C, & D align such that all paths are clear.

Most commonly a Template coming from above is either stopped immediately by the B layer (gray zones), or makes it though B via a 29pipe only to be stopped by a gray square in C, or survives C to be stopped somewhere in D.

The goal here is to understand the obverse situation: the clear channels in block D. The blue clear sections which, like our green sextuplet, have no strikes but cannot connect with the A layer because they are stopped either by block C or block B. When these clear channels form a clear path through both D and C we have a "QPipe".

A sextuplet will be realized whenever we have alignment of a QPipe and a 29pipe, and so any conclusions about the nature of sextuplets depends on the distribution and frequency of QPipes. If a QPipe does not align with a 29Pipe it is a "closed" QPipe and analysis of critical sections by computer shows they are common, even at large n.

6.3.1 The pP(31) filter

Block C, Q_0^{31}, is the gate-keeper between the two realms of primes less than 30 and primes greater than 30. It is the most dense of the potPrimes having 6 strikes in its cycle of 31, meaning only 25 nodes of the cycle are clear to allow sextuplet survival.

All potPrimes have 6 strikes each in their cycles corresponding to which position in a sextuplet template is effected. Each potPrime family larger than Q_0^{31} has a lower density of strike nodes to clear nodes. For example, Q_0^{37} has six strikes out of its cycle of 37 leaving 31 clear nodes. In addition, as n grows each potPrime is inflated in length by 30: Q_1^{37} has 6 strikes out of its cycle of 67 leaving 61 clear nodes. At Q_{10}^{59} the potPrimes length is 359 and it has 353 clear nodes. When n is sufficiently large the majority of the potPrimes at that n and beyond are almost completely clear nodes.

Even though the density rapidly decreases as one moves from the top to the bottom of block D (ie., as n increases), the clear sections must jump the final hurdle of Q_0^{31}'s 25 clear nodes to become a QPipe and so Q_0^{31} has the maximum limiting effect for sexuplet survival.

Despite Q_0^{31}'s "narrowness" it is quite easy to see that there will be infinitely many opportunities for sextuplet templates to survive. Simply multiply the 29basis length by 31: $215656441 \times 31 = 6685349671$. So by the time we've gone that many TNumbers, equivalent to thirty-one 29bases, each position in Q_0^{31}'s cycle has visited each 29pipe in the 29basis; that is, all possible combinations of 31's clear nodes with the 29pipes occur, hence the opportunities are infinite.

6.3.2 Black sections

Critical sections can be "black". I use this term when a critical section has no QPipes; ie., all channels are blocked with a strike. Relating this back to the diagram on page 73: there are no blue channels.

By a limited number of computer analyses it appears black sections are rare. At low n they occur often, but in the higher n's I've looked at I've not found any. This is fascinating and is currently being studied.

Black sections can arise from both non-prime, and prime, potPrime critical sections and so their formation is not dependant on the potPrime's primality.

6.3.3 Questions needing answers for a proof

How often do Q-pipes arise? Is there any way to predict their frequency? If their frequency is high can we be assured that they *must*, at some point, align with a 29pipe? Are black sections infinite or do they die out?

The path to proving, or disproving, the infinite nature of prime sextuplets seems to me, given that Q_0^{31} provides infinite access to 29Pipes as argued above in 6.3.1 on the preceding page, consists of:

- Can it be shown there will always be QPipes?
- Will these QPipes be located in such a way that they are guaranteed to, at some point, have aligned with every possible Q_0^{31} position?

If the answers are yes, then sextuplets are infinite.

The most important question is the first, a QPipe by definition passes the Q_0^{31} filter effect and so, by simply existing, an infinity of QPipes seems to satisfy the already proven infinitely many alignments of QPipes and 29Pipes.

But the second question is subtler. We already know that each Q_0^{31} position will visit every 29Pipe position repeatedly forever. But will each QPipe visit each Q_0^{31} position repeatedly forever? Or will there be some mechanism, at some arbitrarily large n, that blocks off certain positions of the Q_0^{31} filtering pattern?

In other words, let A be the Q_0^{31} filter, B the 29Pipes, and C the QPipes: if A is synchronized with B, and C is synchronized with A, is C synchronized with B? Could it be that the synchronization of A and C are, or becomes, perfectly out of step with the synchronization of A and B?

All in all the QPipes are a tantalizing puzzle.

6.3.4 Static nature of critical sections

Looking again at the diagram one becomes tempted to shift one of the rows in D to see how clear channels disappear and blocked channels open. How do I know this? Because it tempted me.

But resist the urge, after considering this as a possible means of analysis I realized that changing the offset of one or more lines converts the patterns “Gestalt” at that point to one that does not, *and can not*, exist in actuality.

It is analogous to turning a single gear in a watch by hand: doing so moves the other gears too. For example, shifting the Q_0^{37} strike pattern in the diagram to the right by one square actually shifts it to another csID. But another csID has, by definition, a different length and number of rows, and so the shifting has resulted in an impossible critical section.

This is another way of saying that each critical rectangle is a signature, a fingerprint, unique in all the infinity of critical rectangles. Each critical rectangle has completely different combinatorics. There can be no copies of the patterns Gestalt anywhere else in the infinity of numbers, and changing a critical rectangle by shifting a potPrime will change it into a Gestalt that can never exist. Critical rectangles just *“are”*.

I don’t want to mislead you, certainly one can construct a valid critical rectangle, but one must always take into account the state of the offsets of all potPrimes and then generate the pattern from critical sections border using each potPrime’s modular cycle. At low n’s this is feasible, but it quickly becomes computationally impractical as n increases.

6.4 Pattern Combinatorics, artificial potPrimes

This section is under development.

6.5 Strike density

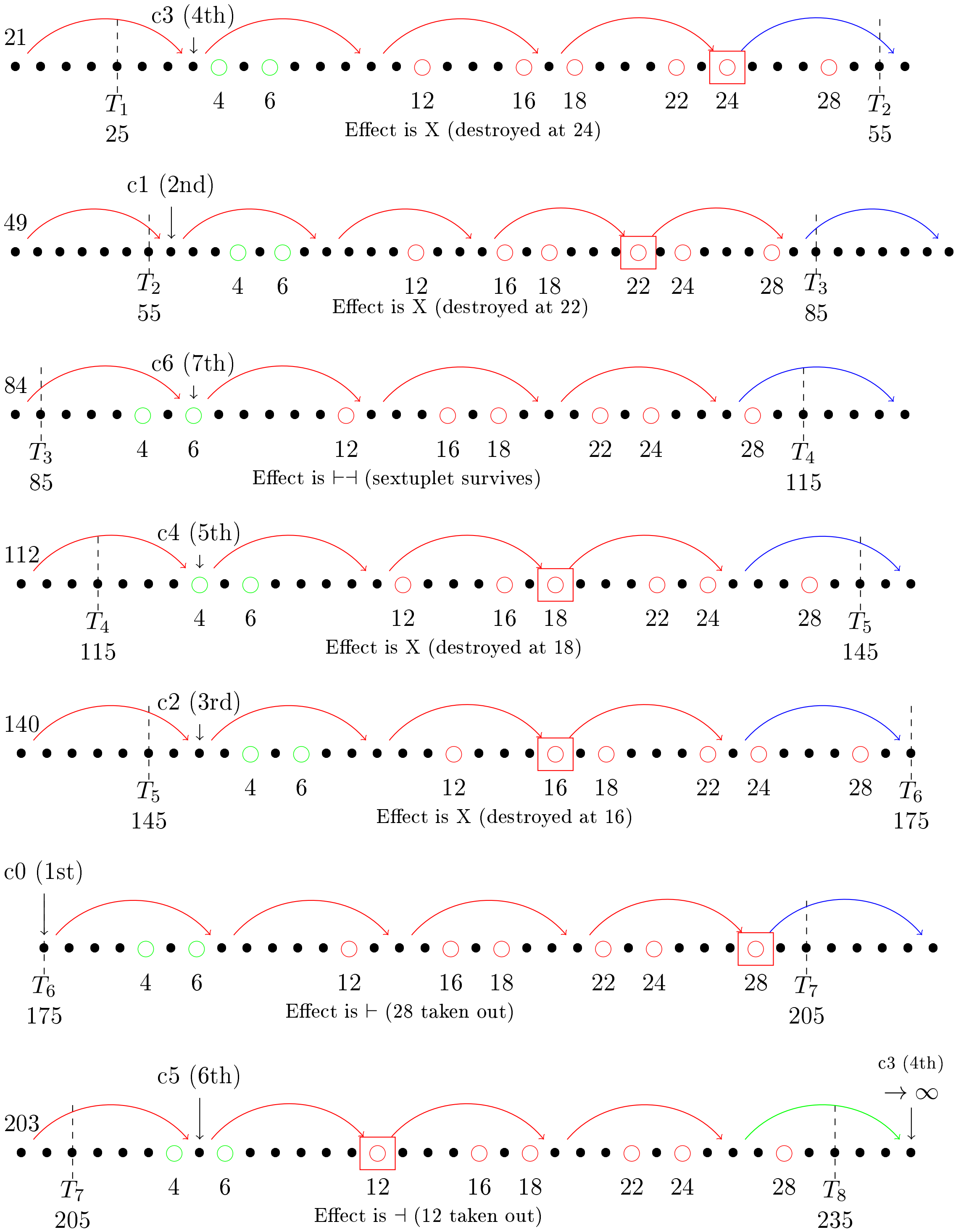
This section is under development.

7 Appendices

7.1 Complete crossing number analysis for prime 7

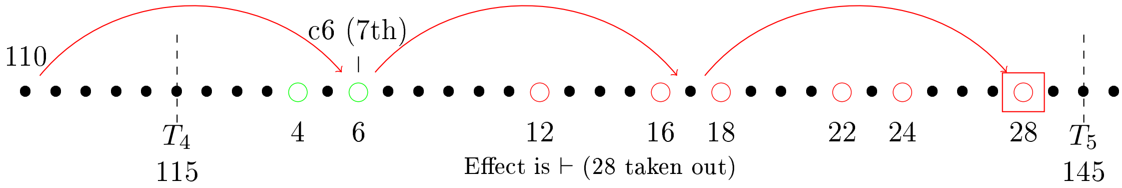
All primes, p, have a repeating pattern with length L against the Prime Template, length 30, which is simply their product $L = p \times 30$. For prime 7 its pattern repeats after 210. This length contains 30 7-blocks, or alternatively 7 30-blocks. Within this pattern we want to map out how 7 strikes out members of the Prime Template and here is how that is done.

Since we start with prime 7’s effective TNumber, T_1, the following diagrams also develop the prime’s natural progression.


21
c3 (4th)
4
6
12
16
18
22
24
28
T_1
25
T_2
55
Effect is X (destroyed at 24)
49
c1 (2nd)
T_2
55
4
6
12
16
18
22
24
28
T_3
85
Effect is X (destroyed at 22)
84
c6 (7th)
T_3
85
4
6
12
16
18
22
24
28
T_4
115
Effect is ⊢⊣ (sextuplet survives)
112
c4 (5th)
T_4
115
4
6
12
16
18
22
24
28
T_5
145
Effect is X (destroyed at 18)
140
c2 (3rd)
T_5
145
4
6
12
16
18
22
24
28
T_6
175
Effect is X (destroyed at 16)
c0 (1st)
T_6
175
4
6
12
16
18
22
24
28
T_7
205
Effect is ⊢ (28 taken out)
203
c5 (6th)
c3 (4th)
$\to \infty$
T_7
205
4
6
12
16
18
22
24
28
T_8
235
Effect is ⊣ (12 taken out)

7.2 Crossing map for 1 cycle of prime 11

A last example diagram showing how prime 11 would be analyzed in a similar fashion starting at its effective TNumber T_4. All other primes 11-29 were analyzed in the same way and their crossing numbers and natural progressions shown in their respective Detail sections. A similar analysis also produces maps for primes 31-59.



7.3 Offsets of preceding effective TNumbers to a chosen N

This is the description of the details from the reference for those wanting to investigate offsets of preceding effective TNumbers at the effective TNumber boundary of a chosen **N** (section 6.1.1 on page 63).

Using equation 24 on page 64 choose the **N** of interest and then any previous n, or iterate from $n = \mathbf{N} - 1 \to 0$, and then setting $d = n - \mathbf{N}$ at each iteration. The offset, o, to that effective TNumber at **N** will be:

$$o = (d\,(2Q_{\text{base}} + Cd + 2C\mathbf{N}) \mod Q_n) - 1 \tag{26}$$

The offsets could be interesting perhaps, but I've yet to find them particularly useful or insightful.

NOTE FOR ANYONE INVESTIGATING THESE EFFECTIVE TNUMBER OFFSETS:

One will notice a seemingly curious pattern in the offsets and to forestall unnecessary work on your part I share these findings:

The larger the **N** one chooses, one sees a seemingly significant outcome that increasingly more of the "bottom" level offsets (i.e., larger n) become constant in this pattern: 29, 119, 269, 479,... and this is true for all Q.

Remembering that $C = 30$, notice that each is equal to $Cd^2 - 1$, for $d = 1, 2, 3, 4$. When $n = 1$ one gets one constant line, when $n = 5$ one gets 2 constant lines, when $n = 11$ one gets 3 constant lines,...

In fact, the number of constant lines is tied to when n is equal to or greater than $\left(\sum_{d=1}^{d} 2d\right) - 1$ at that n, and the number of constant lines is equal to d which becomes, as seen the relationship, the number of terms in the sum.

This is an artifact which may, or may not, be useful in some edge cases, but which I believe at this time has no significance. One can see how this arises by looking at the following construction of the critical equation with d multiplied through:

$$2C\mathbf{N}d + Cd^2 + 2Q_o d$$

The constant amounts are from the Cd^2 term, which increases quickly in size, while the $2C\mathbf{N}d$ term grows more slowly and can only accomodate the Cd^2 value fully when at high enough $\mathbf{N}$. That is, when $2C\mathbf{N}d$ has finally grown large enough such that the $Q_{\mathbf{N}}$ can hold the entire Cd^2 value and can then return that value (minus 1) as the offset.