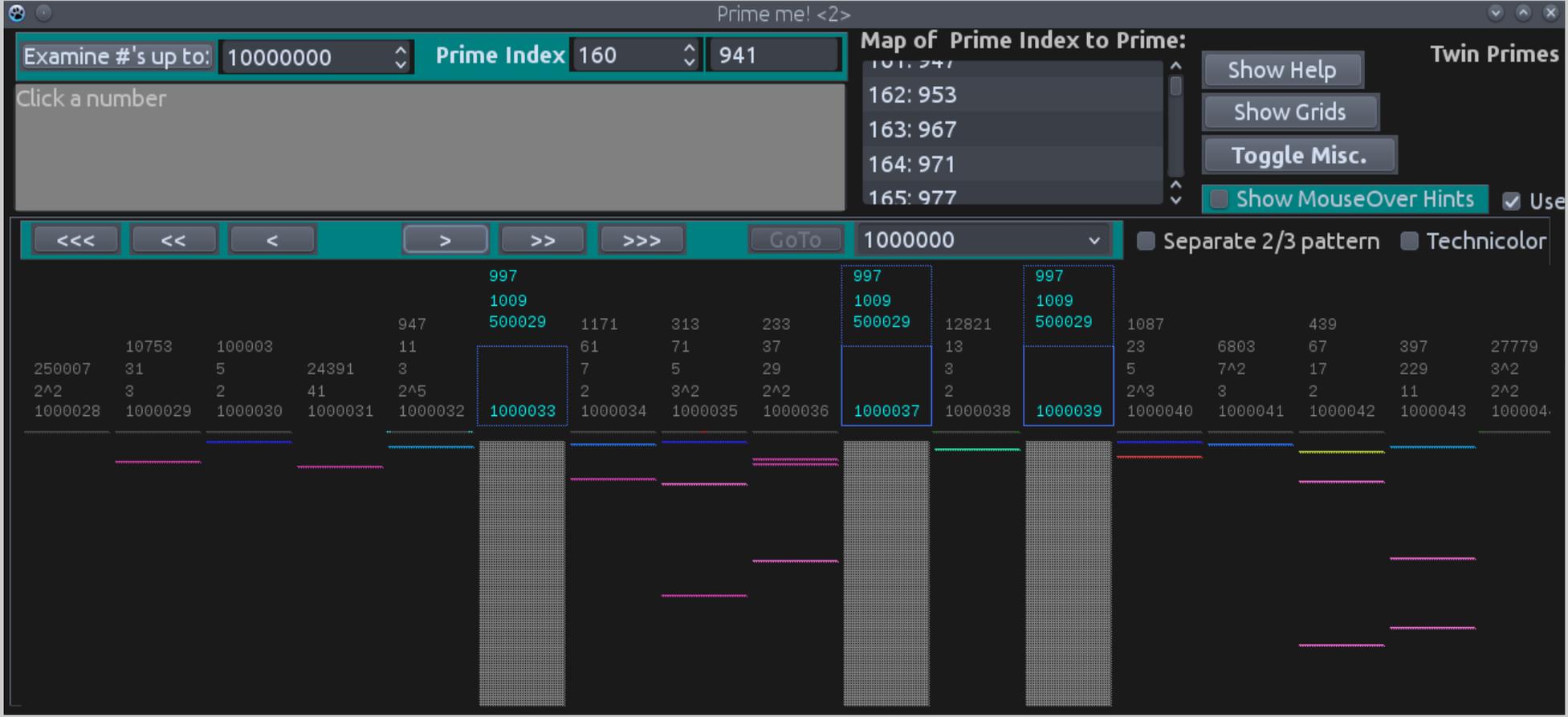


Twin Prime Inspector / Analyzer (License info at end of this document)

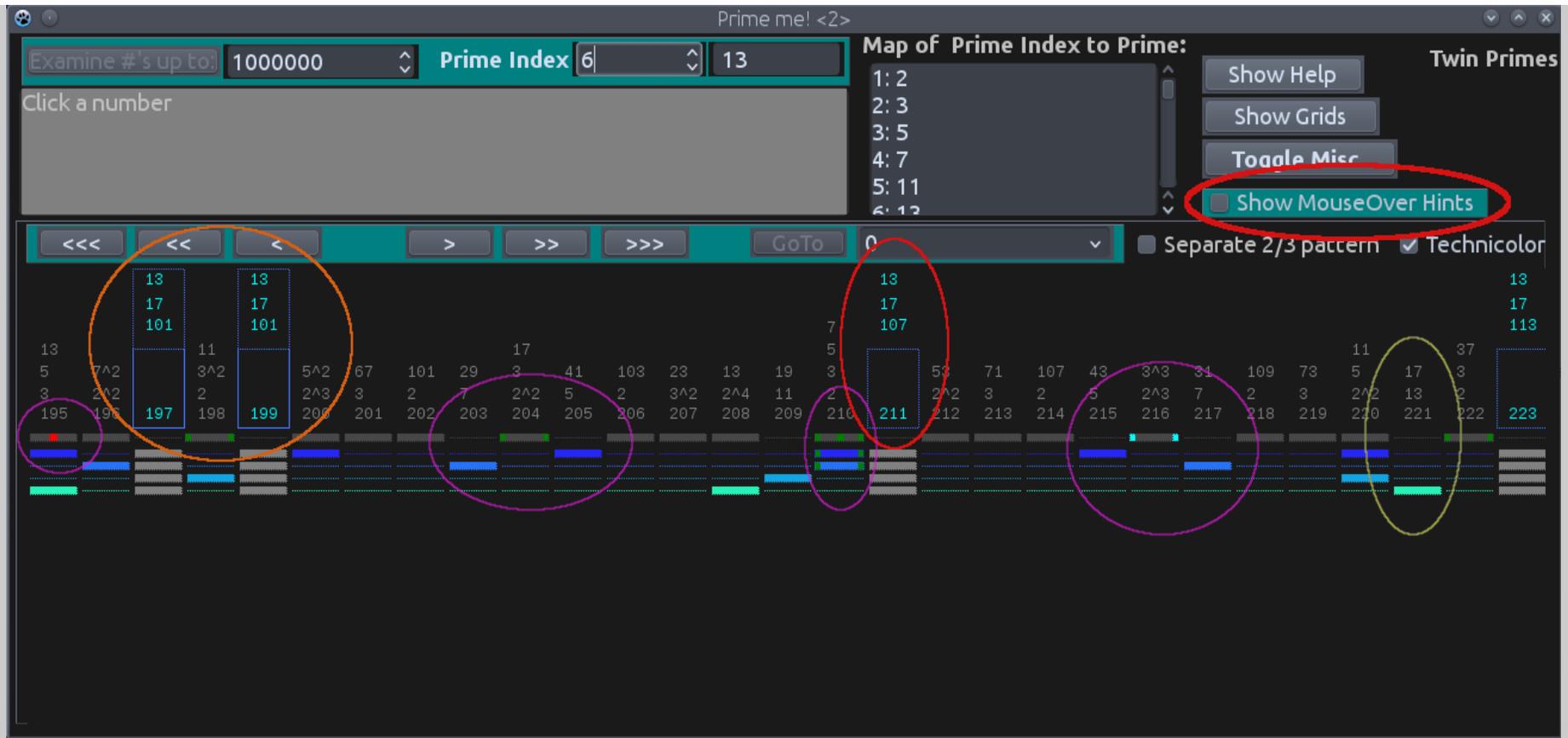
This is a Twin Prime inspector (well, actually, all primes, the focus is on Twins).

The program uses the Sieve of Eratosthenes method to construct the primality of numbers. This allows one to apply Primes to the sieve in a stepwise fashion and examine the results of any particular prime number on the Sieve and to study the resulting patterns, etc. So it like a microscope for Prime Number analysis.

It allows up to 2,000,000,000 numbers to be analyzed and it's pretty speedy. Though it will be a bit slower when you get into the 100,000,000 range, but the wait is not long. By default, at this time, it opens up looking at only the first million (1,000,000) numbers.



On the next page we begin to find our way around, what the symbols and colors mean, and will get into the basic patterns you will see...



First I recommend you click “Show MouseOver Hints” (thick red circle). Then hover over any control to get the details of how that control works. Turn it off when you are done reading as it will become annoying after a while... ;-)

The thin red circle is highlighting the Prime Number 211. It is a “true” Prime (more on that later) and all true Primes are shown in this aqua color. Any time you see a number with this aqua color it is a true Prime. Single Primes are always marked by a half-height blue rectangle.

The Orange circle is highlighting a pair of Twin Primes, 197 and 199. First you know they are true...they are aqua color. To help one discern Twin Primes the blue rectangle is always full height. The numbers over the Primes, also in aqua, are landmarks associated with this Prime Number. The first is the Prime Number who's square “clears” this region of the number line (more on clearing later), the next is the Next Prime Number after the first listed one and who's square marks the end of this “cleared” region. And the third is the Prime Number that is greater than half this current Prime Number. When you left click a Prime the statistics for that Prime, generated by these numbers, will be displayed in the grey text area. These statistics will make more sense later once we've looked at specific regions of the number line.

For example, in Prime 211: 13 is the Prime who's square ( $13^2 = 169$ ) is less than 211. 17 is the next Prime after 13 and it's square ( $17^2 = 289$ ) and this let's you know that the pattern existing at this point will be changing at 288. 107 is the Prime Number who's value times 2 ( $107 * 2 = 214$ ) is greater than half of this Prime Number ( $211 / 2 = 105.5$ ). Again, these statistics will make more sense later on...

The Purple circles introduce us to the “basic” Pattern of Primes (we will see this in detail in a bit). Notice under the primes the first line is made up of dark grey rectangles, sometimes they have color in them. These grey rectangles are THE basic pattern of Primes, and they are so important that they have a code to help you know where you are in relation to the Basic Pattern. First notice that 198, 204, 210 and 222 all have dark green dots on either end: this means that number is divisible by 6. 216 also has dots but they are a brighter green, this happens every 4 repetitions of the Basic Pattern and it means that number is divisible by 24 (an important number!, you'll see...).

Now notice 195. It has a red dot in the middle of the grey rectangle of the Basic Pattern. This means that number is divisible by 15, also a very important number. And 210 has a green dot in the middle...that means this number is divisible by 30, also important.

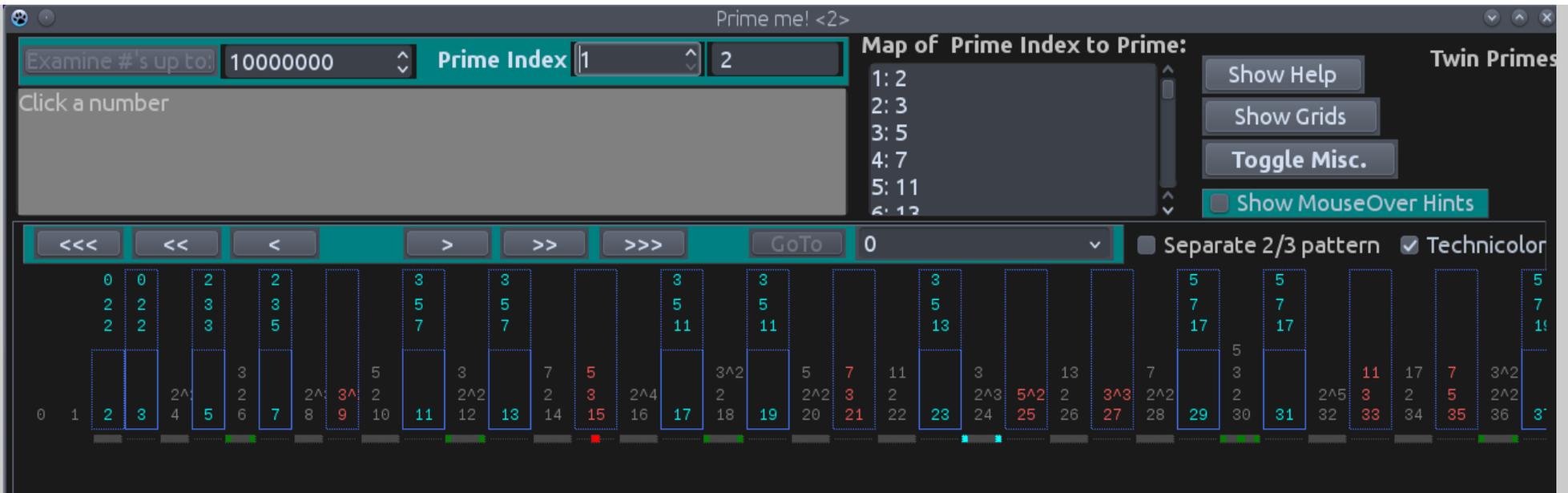
Finally in our introduction is the yellow circle around 221. Notice in the Basic Pattern of grey rectangles that there are regular voids / spaces. It is ONLY in these voids / spaces where Prime Numbers CAN BE. Notice that the True Primes in this view (197, 199, 211 and 223) occur in these voids, they have to as I said. But there are other voids, as you can see, and they are not Primes (203, 205, 209, 215, 217 and 221, also notice that numbers that are not Prime are grey in color). That is because they are divisible by Prime numbers (Prime Factorization) and so, by definition, are not Prime. First notice how the figures (also in grey) are stacked over the number differently than how they are by Primes. Those numbers are the Prime Factors of that number. In the case of 221 it is the product of  $13 * 17$ . Now look below 221 and notice all spaces are voids until the bright green rectangle. The bright green rectangle is showing the pattern of the Prime number 13 and so in this way one gets to see at a glance how the Prime Factors interact to allow, or disallow, a particular Basic Pattern void to be Prime.

So each row you see below all the numbers shows the pattern of successive Prime numbers for that region. As already mentioned the first row (dark grey rectangles) is the Basic Pattern (it is a combination of the patterns the Prime Numbers 2 and 3, we'll see this in a moment). The next row is the pattern of 5, the next the pattern of 7, then next of 11, and the last of 13. There is not enough room to write this information but it is easy to determine which rows and rectangles are which Prime Numbers by simply looking up at the Prime Factors for that number, this will tell which Prime Numbers are having their effect at this point. For Example: Take a look at 208 and 195. They also both have a bright green rectangle in that row just like 221 does. Examine their Prime Factors...they are all divisible by 13. 208 is  $2^4 (= 16) * 13$ , and 195 is  $3 * 5 * 13$ .

Number 210 is the product of  $2 * 3 * 5 * 7$ ...2 & 3 are combined in the Basic Pattern, and there is a rectangle in each of the 5 row and 7 row underneath. This allow one to visually see the patterns that are running through the Prime Numbers and number line.

Ok, that's it for basics. Below is the same Program view but without all the colored circles. Study it and put your new found understanding to the test....





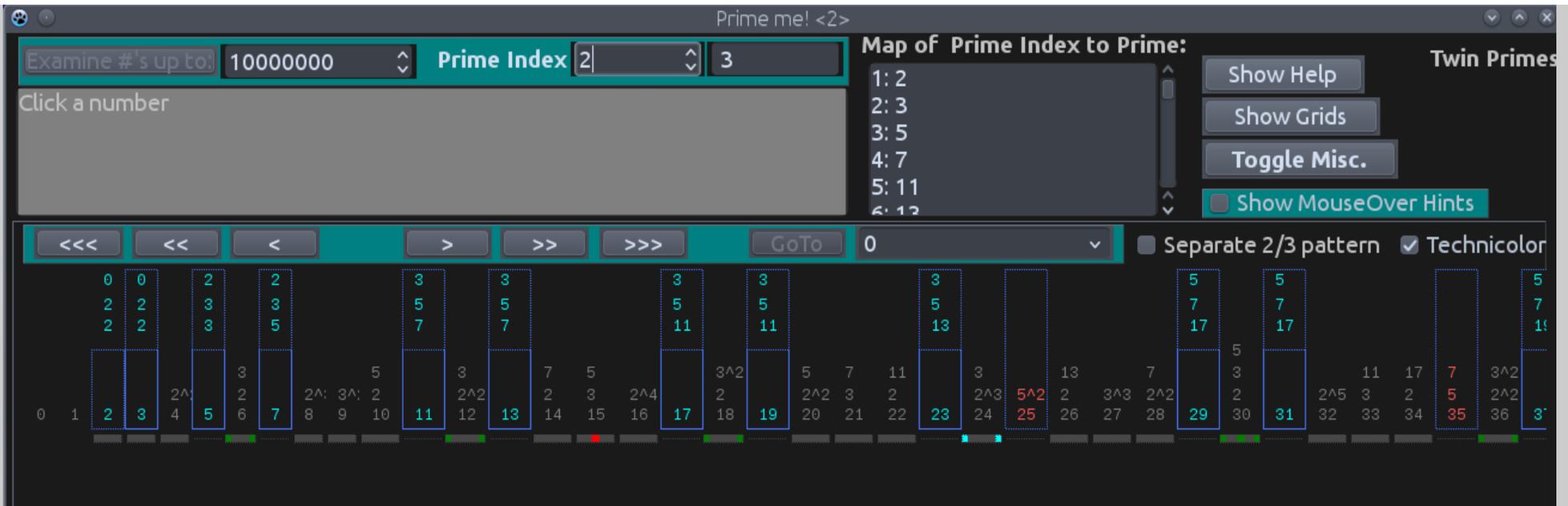
Here you see the most basic situation for the Sieve. It is the number line with Prime Index 1 (P=2) applied. So we are ONLY applying the Prime Number 2 at this point. The Sieve is that we start at 2 and then remove every 2<sup>nd</sup> number. The result is that all odd numbers are considered Prime! Why would 9, or 15, or 21 be considered Prime, we know they are not Prime.

Because we've only applied the first Prime Number. This is the entire point of this program: to stepwise apply the Sieve of Eratosthenes and see the effects. On the next page we will apply Prime Index 2 (P=3) but first a word about...

...the red numbers. This is the first time we've seen them. As you've by now surmised numbers in red are "false" Primes. As far as the application of Prime Number 2 is concerned every odd number is then Prime...but some are "true" and some are "false". So you see at any level which numbers are considered Prime at that point but under the cover the program does the math and indicates which ones are true and which ones are false at that point. The underlying construct of these views is always done using the Sieve method, the trueness or falseness is calculated after the fact to help one see what is going on at each stage of Prime Number application.

Notice in this view that each Prime, whether true or false, is considered to also be a Twin Prime (tall blue rectangle around it). Again this is due to the fact that we have only applied the Prime Number 2, and since each odd number is considered Prime, and since each "Prime" number is 2 away from it's neighbor they all are Twin Primes, at least as far as Prime Number 2 is concerned.

This will change quickly. Very quickly...



Notice in this view we have applied Prime Index 2 (P=3) and it is already much, much cleaner. The 9, 15, 21... and so on are now removed from the Primes because all are divisible by 3. Notice 25 and 35 remain "false", we haven't applied Prime Number 5 yet...

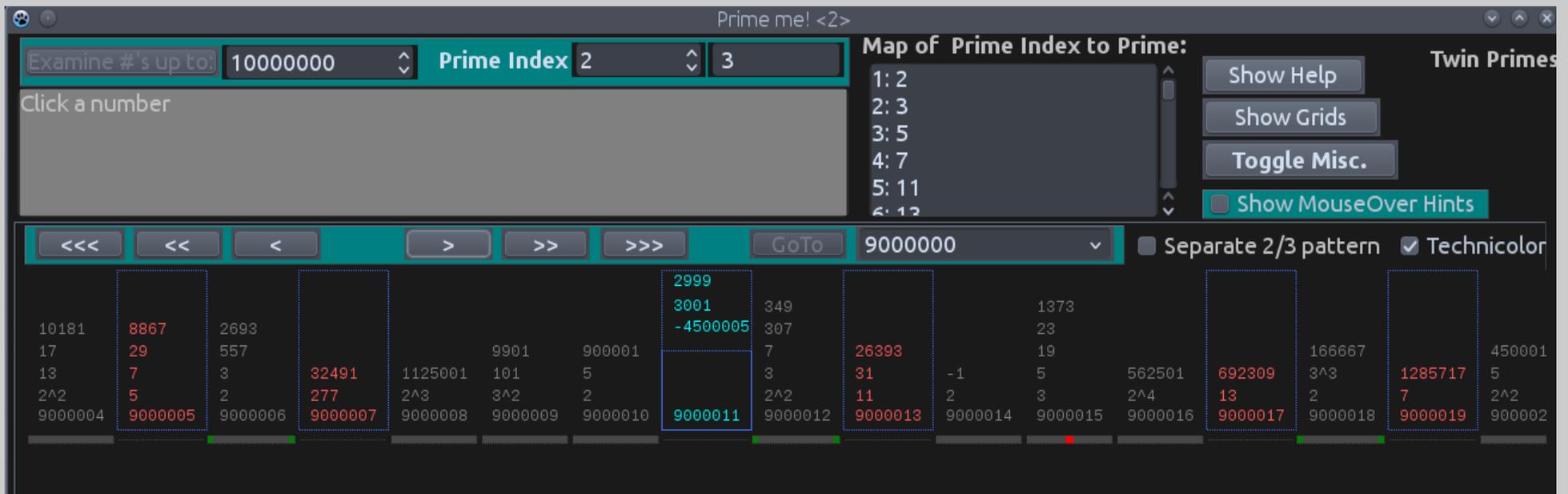
Even though "false" Primes are considered to be Primes at this point, the program treats them like composite numbers (well, because they actually are) and displays, also in red, the Prime Factors for that false Prime. This lets you know when that false Prime will be cleared. In the picture above both false Primes are divisible by 5 and so they will be removed from Prime-ality when we apply Prime Number 5.

After this application of Prime Number 3 we have arrived at the "Basic Pattern" of the Primes. Ignoring 2 and 3, the pattern is always Twin Primes separated by 3 non-primes...to infinity. It is this pattern that allows Primes to occur and when the other Primes interact with this pattern they can leave these Prime gaps free (then the number truly is Prime) or they can block it (indicating the number can't be Prime at that position).

As discussed before the first row of dark grey rectangles summarizes this Basic Pattern. It is based on the interaction of 2 & 3 and so is a repeating pattern of 6 possibilities. The first space is always blank and allows Prime-ality, the 2<sup>nd</sup> is always divisible by 2 and so blocks Prime-ality, the third is always divisible by 3 and so blocks, the fourth is always divisible by 2<sup>2</sup> and so blocks, the fifth is always free and so allows Prime-ality, and the sixth is always divisible by 2\*3. Then the pattern repeats...forever. Examine the line above now to see this...

The pattern always ends on a number that is divisible by 6, look at 12 for instance. The Pattern starts anew at the following number, here 13 and 13 is Prime, then 14 (div. By 2) 15 (div. By 3) and 16 (div. By 4) are blocked. Then the 17, the fifth position of the pattern is Prime, then we are at 18 and the pattern is finished.

This pattern literally controls all primes! Notice that Twin Primes CANNOT be in the middle of one of these patterns. Twin Primes are formed by successive patterns where the 5<sup>th</sup> position of a pattern must interact with the 1<sup>st</sup> position of the next pattern.

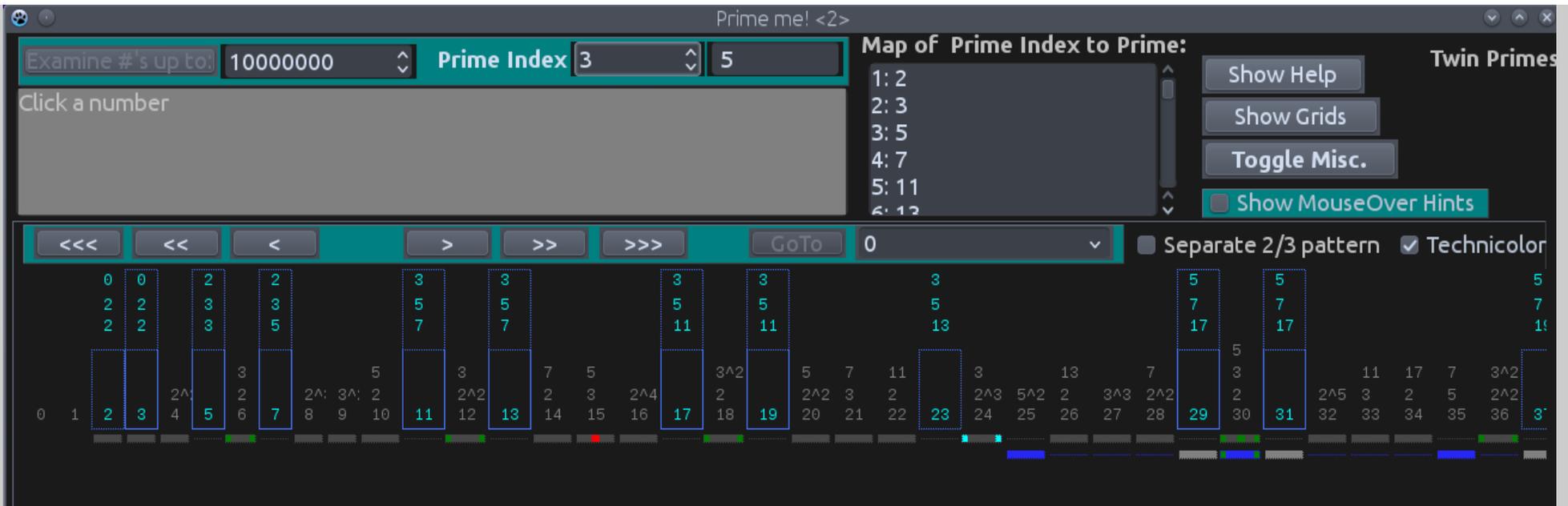


Here for your enjoyment is view of the Basic Pattern starting at 9,000,004 with just Prime Numbers 2 & 3 applied. It just repeats forever! Twin Prime, followed by 3 non-prime numbers then another Twin Prime, to infinity. Here, in this range you can see that most of them are false. Almost all of them will be cleared quite quickly. By the time we've applied Prime Number 13 all will be gone except for false prime 9,000,007. That one will continue to be false Prime until Prime Number 277 is applied (look at it's Prime Factors).

Note again: only numbers at the 5<sup>th</sup> and 1<sup>st</sup> positions of the Basic Pattern “can” be Prime...whether they are true or not depends on how the other patterns of Prime Numbers interact with the Basic Pattern, and that will be seen as successive Prime Indexes are applied.

One other thing to note here. In the True Prime 9,000,011 the 3<sup>rd</sup> statistic number is an approximation. That is because I don't store enough Primes in the program to actually be able to calculate the Prime Number that is more than half of this number. I only store the Primes that you can find in the Map of Prime Index to Prime listbox, and that only contains the first 114,175 Primes (all calculated by this program by the way). The 114,175<sup>th</sup> Prime is 1,500,269, half of 9,000,011 is somewhere around 4,500,005. Since I don't have primes up to 4,500,005 stored in the program anything outside of this range is approximated. Just FYI.

One other note: These primes are generated by a Sieving process, not by mathematics. The Prime Numbers are stored simply to do math after the fact to ascertain false Primes' Prime Factors. I've not found any errors in this program and that is due to it not being dependant on my mathematics, but on the Sieve of Eratosthenes which is mechanical and absolutely trustworthy.



Now we go back to 0 and look at the pattern after Prime Index 3 (P=5) is applied. 25 and 35 are no longer false Primes.

Also notice that only now has a second row below been started for the Prime Number 5 (the blue rectangles).

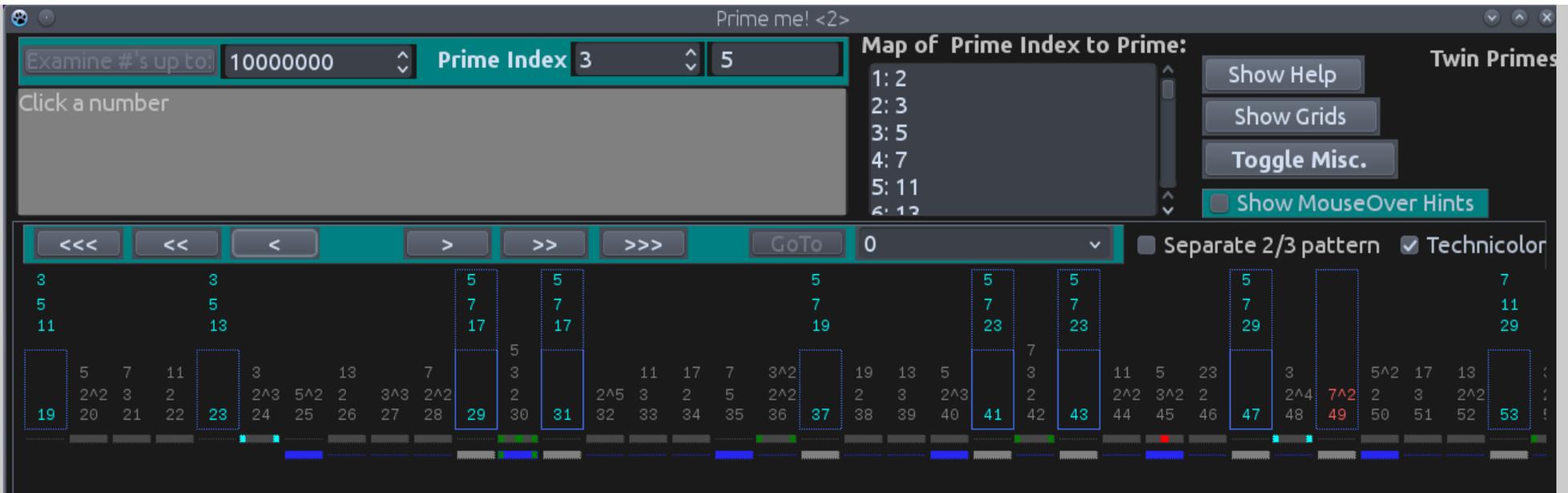
This introduces the concept of the squares of the Primes. We don't start showing a pattern for 5 until its square (25) because 5 HAS NO EFFECT on the patterns until it's square! So there is no need to see it before that.

Yes, 10, 15, and 20 are divisible by 5, but they were divided out before 5 was applied by 2, 3, and 4. 5 can have no effect until its square is reached. This is true for all Primes. And so no new lines are started below until that Primes' square is reached. So, if you are in a region and you increase the Prime Index and no new lines are added below, it is because all the Primes that have an effect in that region have been applied and there is nothing more to show.

This also explains why I've gone to the trouble to note above each true Prime the first two numbers. They indicate which Primes “control” this region. For instance 29 has the numbers 5 and 7 above it. Remember, these are the Primes who's squares control the region (again, clicking on the Prime will give those stats in the grey memo box above). So 29 is controlled by 5 squared and 7 squared; that is, it is between 25 and 49.

Why is this important? Because within these regions the underlying patterns that determine Primes in this region DOES NOT CHANGE, it is constant...up to the next Prime's square. At that point the the next Prime will add itself to the underlying, entwining pattern and the pattern will be changed. Again, this is true for every Prime's square.

In addition when a Prime is applied it “clears” out all false primes in the region between it's square and the next Prime's square. Said another way, once the Prime is applied, all numbers in the region between the squares of those “controlling” numbers have been completely decided as to what is Prime and what is composite. The region is “cleared”.



Confused I bet! Here's an example: Here we've applied Prime Index 3 ( $P=5$ ) and we now look farther up the number line. The square of 5 is 25. The next Prime Number after 5 is 7. The square of 7 is 49. Now examine the region above between 25 and 49. There are no false Primes UNTIL 49 because it is  $7*7$  and not divisible by 5, so it shows up as a false Prime.

Once a Prime has been applied it clears the region, completely, between it's own square and the next Prime Number's square. This is always true for all Primes and their squares.

For people seeking solutions to Prime problems it seems this would be a good fact to keep in mind, it defines a region where nothing in the underlying pattern changes. And keep in mind as the Primes get bigger and bigger these regions keep getting bigger and bigger too. The size of the region between 25 and 49 is only 24 numbers big. But this grows as the size of the Prime Numbers grow and gets bigger and bigger. But!...the regions between Primes squared will always be a size that is a multiple of 24 (after 25, region between 2 squared and 3 squared is 5 in length, and region between 3 squared and 5 squared is 16) ...

That reminds me. Remember that the Basic Pattern always as 2 green dots on either end? And sometimes the green dots are brighter? Compare the green dots on the Basic Pattern under 24 and 48 with the patterns under 30, 36, and 42.

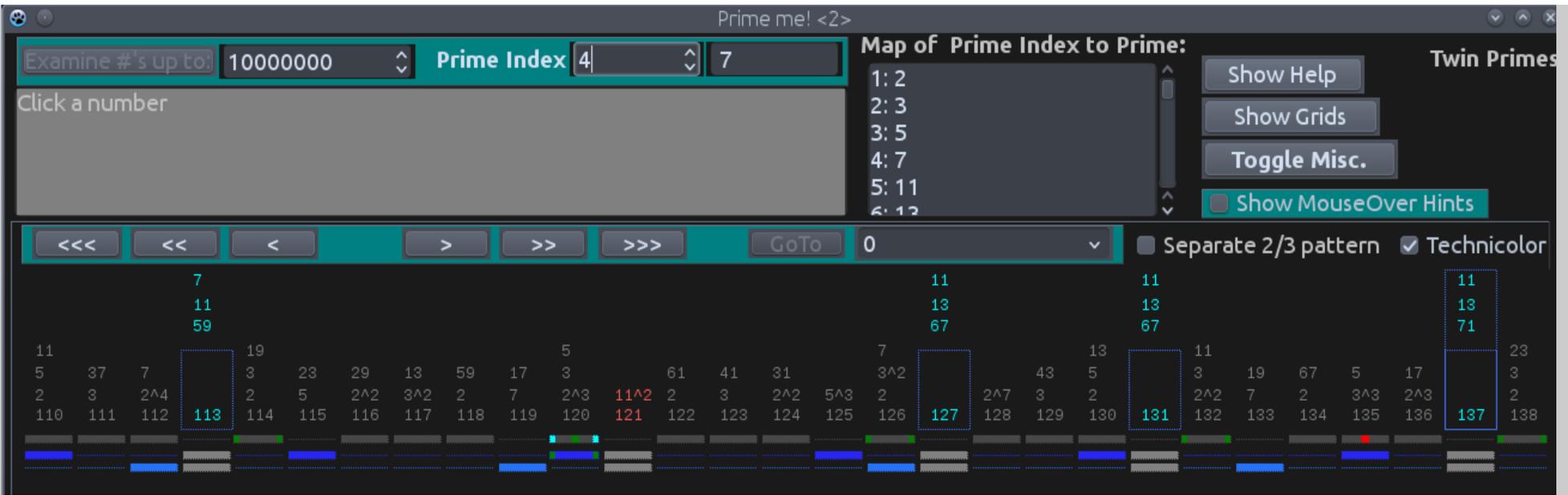
Remember that the bright dots indicate the number is divisible by 24? That is why I've marked them so because of the simple fact that EVERY square of a Prime is of the form  $24n + 1$ . For instance,  $25 : 24(1) + 1 = 25$ . Or  $49: 24(2) + 1 = 49$

Now this does not mean that every number that is of the form  $24n + 1$  is a square of a Prime: for instance  $24(3) + 1 = 73$ . 73 is not a square of a Prime. But if a Prime is squared it WILL BE of that form. So it so important that is why it marked out for you with the bright dots every 24 numbers (equal to four of the Basic Patterns, which are 6 numbers in length).

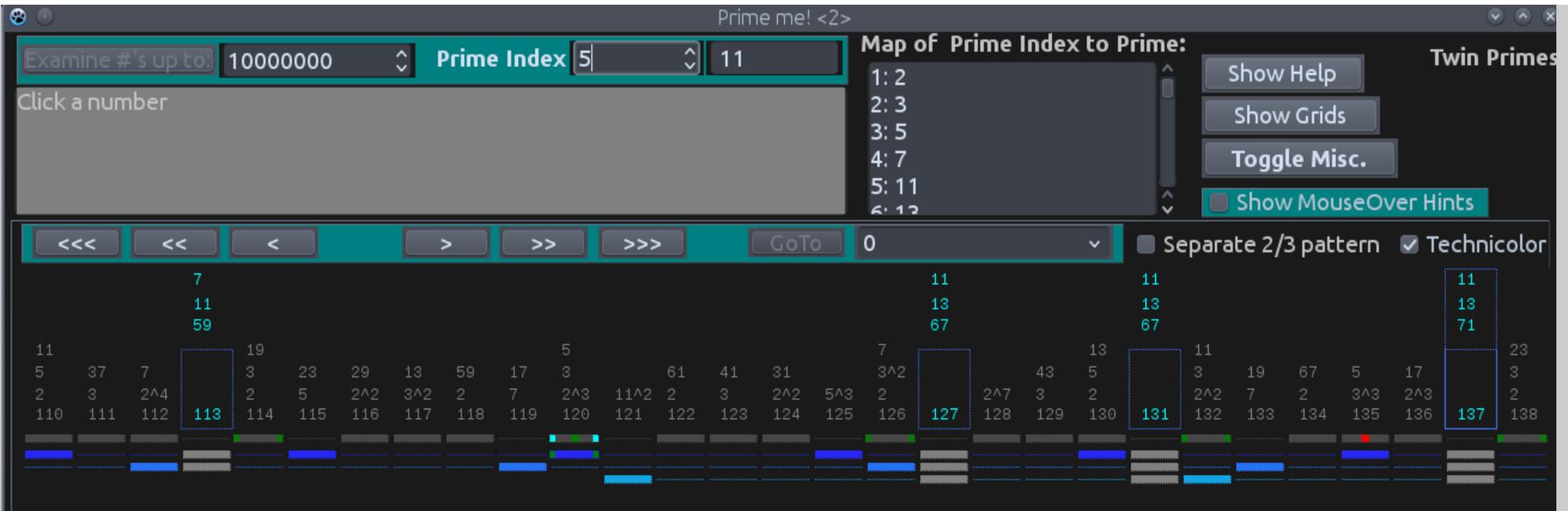








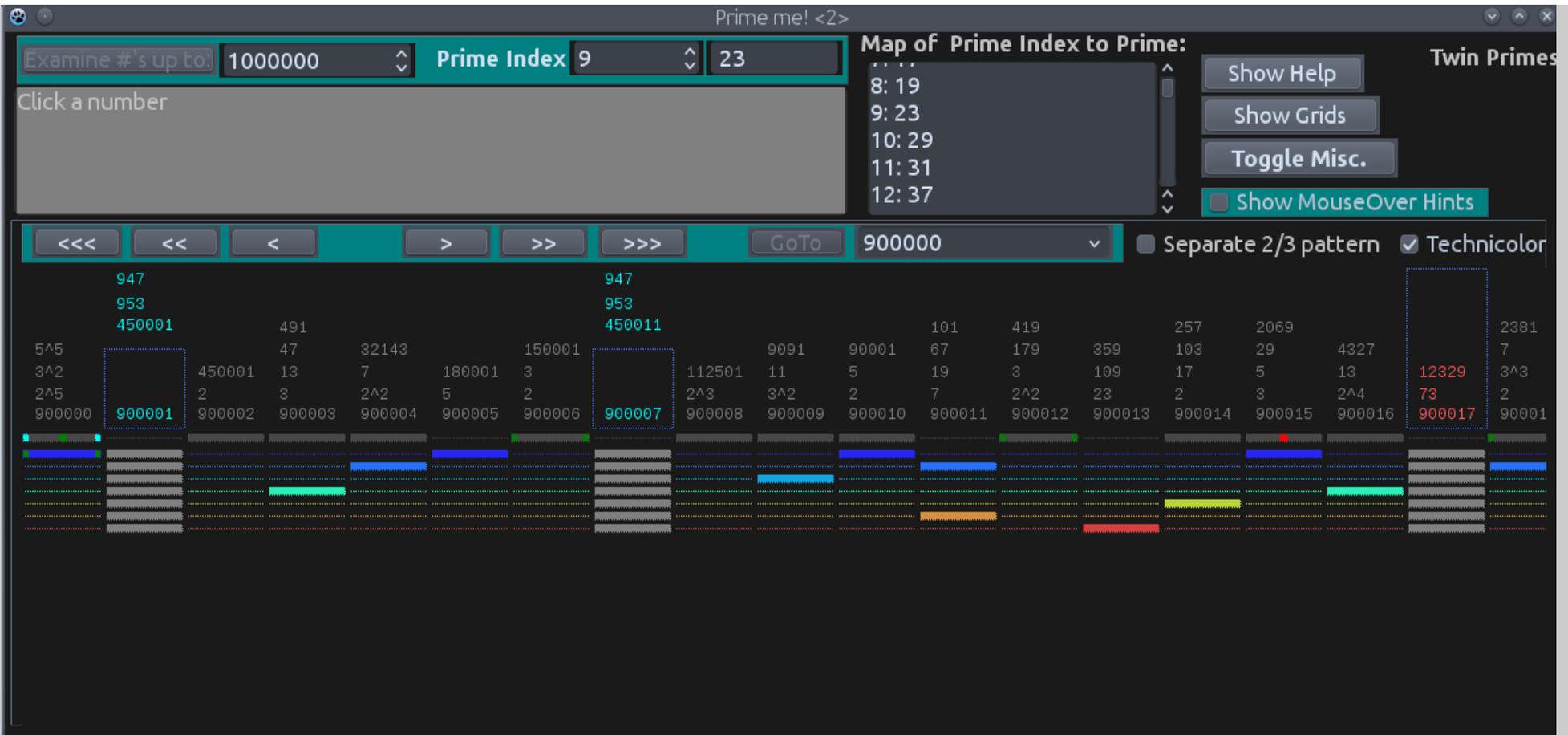
Now we've applied Prime Index 4 (P=7) and all the region from 49 to 120 is completely cleared...



And finally Prime Index 5 (P=11) is applied and this will be cleared out to 168 with 169 being false (13 \* 13).

Two more sections and then this help file will be done...Let's go out to larger regions, how about 900,000....





900,013 falls as a Prime with the application of Prime Index 9 (P=23)...







To illustrate this we will look at the first occurrence of a sextuplet Prime!...

Here we see, with Prime Index 2 (P=3) applied, it shows a complete 24 cycle (bright dots, remember?) from 96 to 120. This is The Pattern of the Primes...but wait! There are actually two sub-patterns here...

Remember red dot in the middle (see 105)? That is a number which is evenly divisible by 15 (3 \* 5). Notice the symmetry around this number.

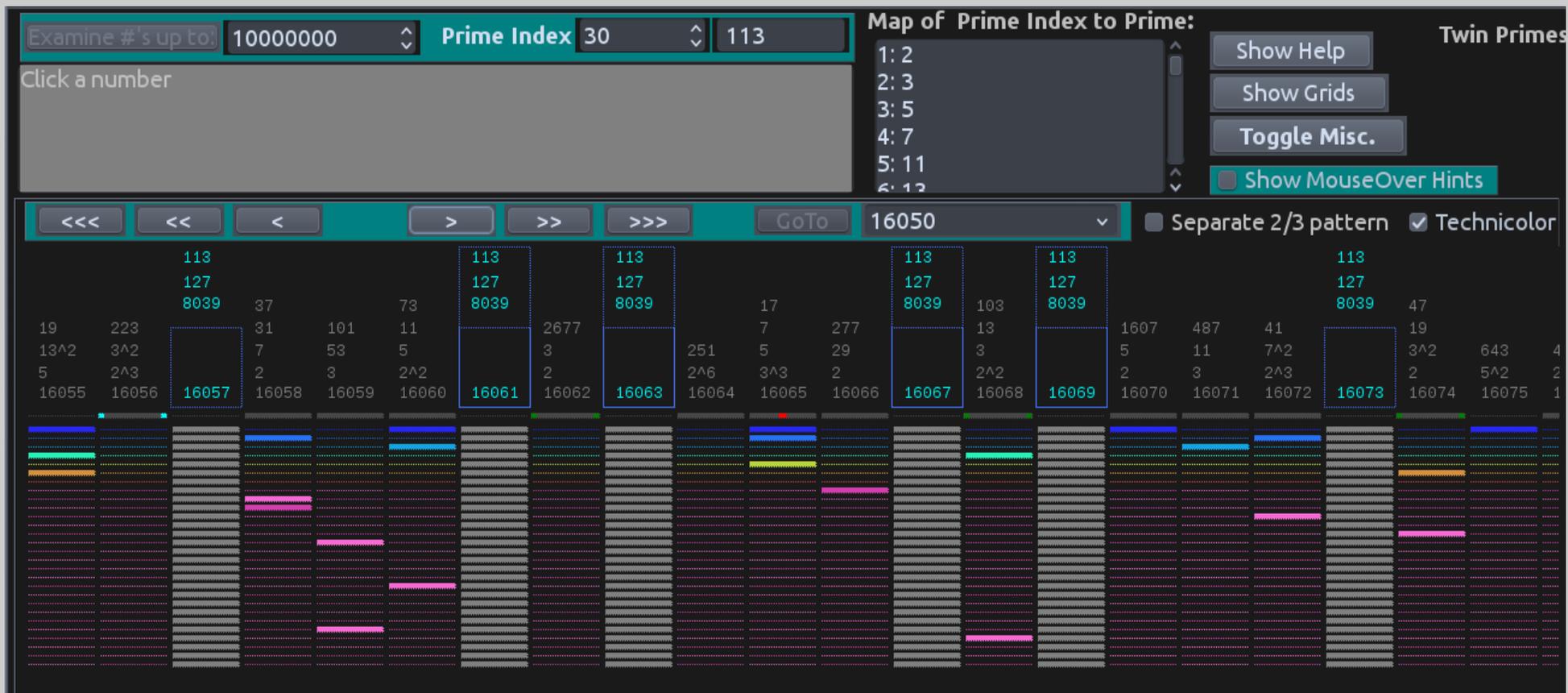
Let's apply Prime Index 3 (P=5)...



Now the "15" pattern is clear! A prime (97) then 3 non-primes, then a Twin Prime, then 3 non-primes, then another Twin Prime, then 3 non-primes, then a Prime. This is a sextuplet. One can also have quintuplets (one of the beginning or ending primes is Sieved out), and quadruplet Primes (where just the the two "Twin" Twin Primes are left. This Pattern is THE pattern and is the most dense you can get in any region of Primes. Here all the numbers fall such that the sextuplet is preserved. Other "15" regions generate many Twins, quads, etc. depending on how the other Primes interact / interfere with this Pattern.

The other sub-pattern is seen at 120. It has a green dot in the middle indicating it is divisible by 30 (2\*3\*6). This is the only other possible position for a Twin Prime (in this view they are both false Primes). Of course sometimes one or the other is crossed out (or sometimes both, like here) and one gets only a single Prime instead. But these are THE Positions where Primes, if they are going to appear, must appear.

That's it. For fun we look at the next occurrence of a sextuplet Prime...



...It is centered around the number 16,065. As you would expect sextuplet Primes are pretty rare, it is not known if there are an infinite number of them. Quints and Quads happen more often as you would expect, but are also rare, and also not known if infinite numbers of them exist. Twin Primes you see EVERYWHERE, they just keep popping up. But again, it is not known if Twin Primes are infinite in number...

So...have fun, hope you figure it out!

If you want, read a couple more pages to see how the Grid works...

This is a Play Area. You can move prime numbers around and see the resulting patterns. Simply left click a line to move it to the left, and right click moves to right. The next to last row is fixed: it is the basic 2/3 pattern and serves as the anchor for the grid and shows where Primes CAN occur. The last row shows the "sum" of the changes you made.

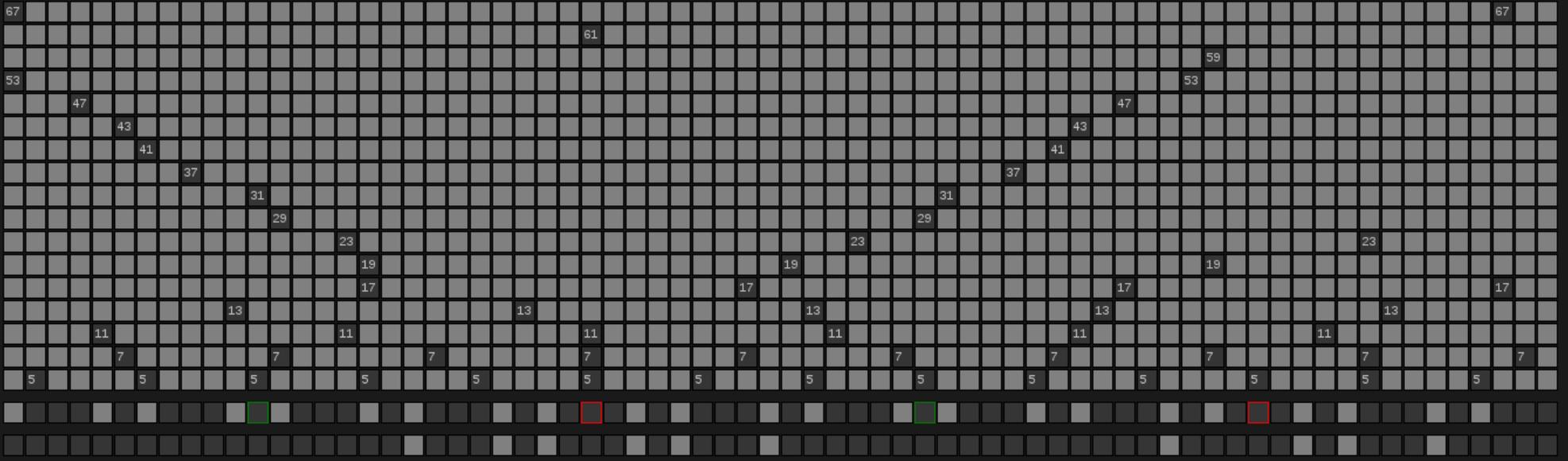


This is the Grid. It allows you to play with the Prime Numbers (up to 67, so it is based on the first 70 positions after 4,489 [67 squared]) and see how the patterns change.

Each grey row is labeled with the Prime Number it represents. You simply left click a row to move it to the left, or right click to move it the right.

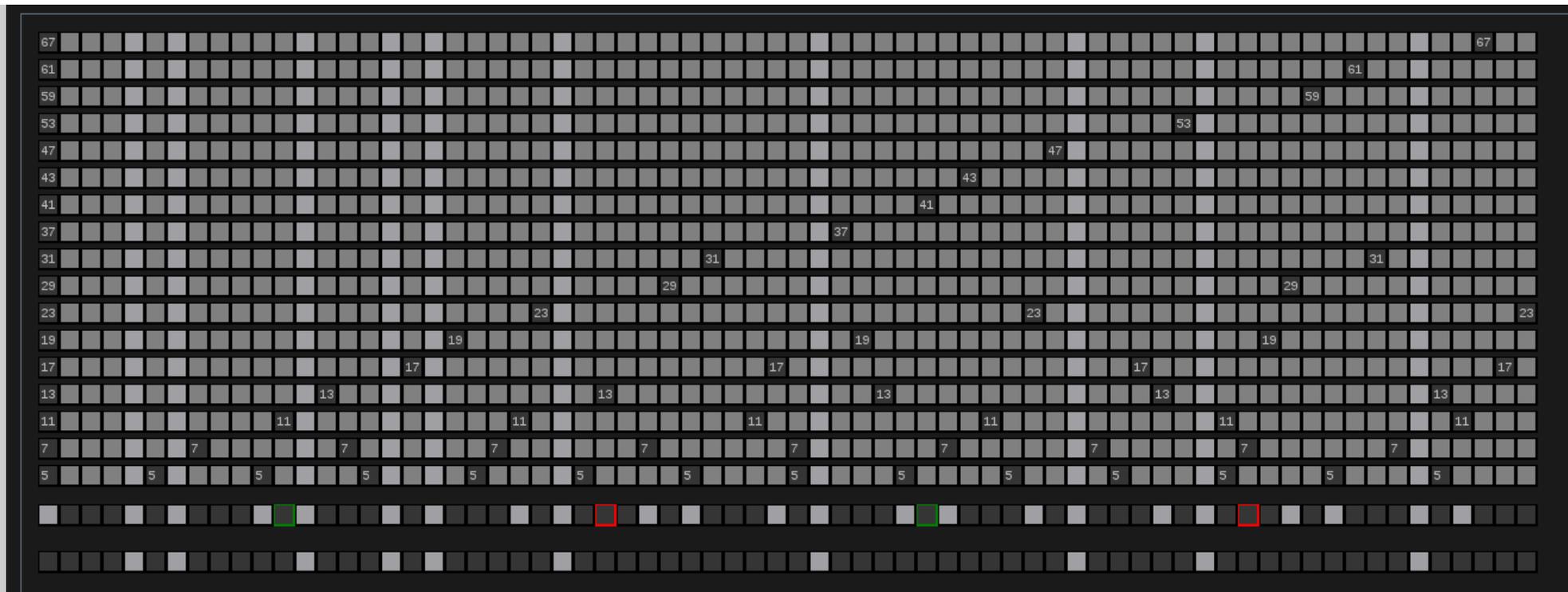
The next to last row is the basic 2/3 pattern, it can not be moved, it serves as the anchor and the bright gray squares show where Primes are allowed to occur. The red outline is as usual a position that is divisible by 15, and the green rectangle is a position divisible by 30.

The last Row shows the summary of all the patterns above. The squares there are only light gray IF that column has no Primes in it. An example....



Here I've moved everything to clear the red square to allow a Prime Sextuplet, you can see in the summary row (the last row) that Prime sextuplet is there. You use this grid to test theories or just for fun.

One last example...



Here I've put every prime in the first column. This is just to illustrate that any pattern you can come up with WILL occur at some point in the Number Line. But you can not assume that even though the pattern here indicates prime numbers (in the summary row) that it will be true. This grid is simply a play area to observe what happens generally (up to 67).

The problem is that where this pattern occurs will be a HUGE number. The first column will be the product of  $5 * 7 * 11 * 13 * \dots$

Were you to go look at this region where this HUGE number occurs there will have been many many more primes that affect it and could potentially (or better probably) wipe out the prime pattern you generated here. For instance adding the Prime Number 71 could wipe out one of these primes, 73 could wipe out another.

Anyway, it is for play, maybe you discover something!

This program is written in Pascal in Lazarus' Rapid Development IDE.

The source code is at: <https://github.com/Juuliusus/TwinPrime>

The original executable is compiled for Linux. If you just want to share the executable be sure to also share this help file: TwinPrimeHelp.pdf

Copyright (C) 2015 Julius Heinrich Ludwig Schön / Ronald Michael Spicer  
created by Julius Schön / R. Spicer  
Foto.TimePirate.org / TimePirate.org / PaganToday.net

The code in this file is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License (GNU GPL) as published by the Free Software Foundation, either version 3 of the License, or (at your option) any later version. The code is distributed WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU GPL for more details.

As additional permission under GNU GPL version 3 section 7, you may distribute non-source (e.g., minimized or compacted) forms of that code without the copy of the GNU GPL normally required by section 4, provided you include this license notice and a URL through which recipients can access the Corresponding Source.